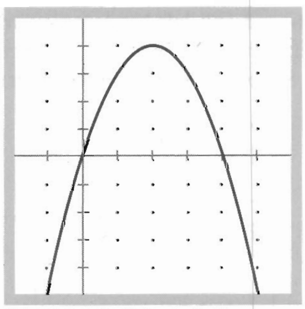
Solution There is no derivative at the origin because the right-hand and left-hand derivatives are different there. The slope at $x = 0$ of the parabola on the left (Fig. 3.11) is $m = 2(0) = 0$ (from Example 3). The slope at $x = 0$ of the line on the right is 2.

Examples 6 and 7 are examples of continuous functions that are not differentiable, counterexamples to the converse of Theorem 1.

Exercises 3.1

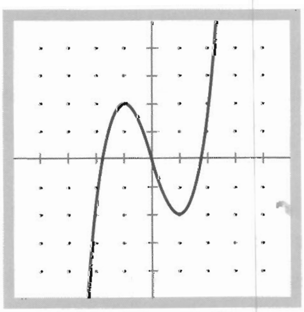
In Exercises 1–4, estimate the slope of the curve (in y-units per x-unit) at the point with the indicated x-coordinate. Be careful: x-scale and y-scale may not equal 1 in the viewing window shown.

1. a) $x = 2$ b) $x = 4$



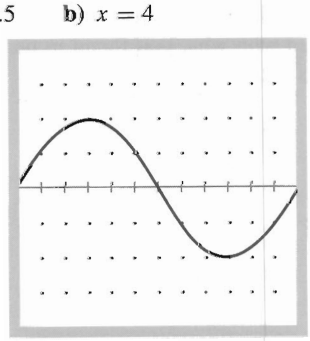
[−2, 6] by [−5, 5]

2. a) $x = -1$ b) $x = 0$



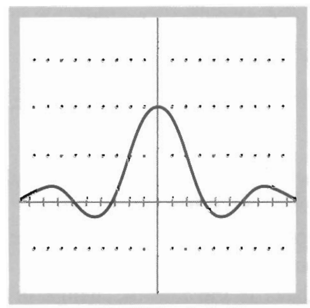
[−5, 5] by [−5, 5]

3. a) $x = 0.5$ b) $x = 4$



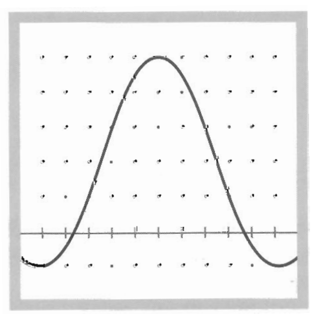
[0, 6] by [−2, 2]

4 a) $x = -1$ b) $x = 1$



[−10, 10] by [−1, 2]

5. The viewing window below shows the Fahrenheit temperature in Fairbanks, Alaska, for a typical 365-day period from January 1 to December 31. Answer the following questions by estimating slopes on the graph in degrees per day. For the purpose of estimation, assume that each month has 30 days.



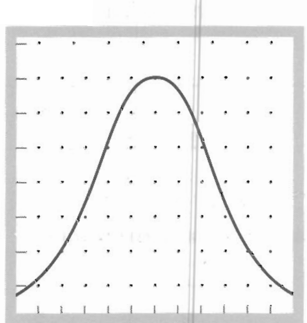
[0, 365] by [-20, 70]

a) On about what date is the temperature increasing at the fastest rate? What is the rate?

b) Do there appear to be days on which the temperature's rate of change is zero? If so, which ones?

c) During what period is the temperature's rate of change positive? Negative?

6. The viewing window below shows the number of hours of daylight in Fairbanks, Alaska, on each day for a typical 365-day period from January 1 to December 31. Answer the following questions by estimating slopes on the graph in hours per day. For the purpose of estimation, assume that each month has 30 days.



[0, 365] by [0, 24]

a) On about what date is the amount of daylight increasing at the fastest rate? What is that rate?
b) Do there appear to be days on which the rate of change in the amount of daylight is zero? If so, which ones?
c) On what dates is the rate of change in the number of daylight hours positive? Negative?

In Exercises 7–20, use Eq. (1) to find the derivative $dy/dx = f'(x)$ of the function $y = f(x)$. Then find the slope of the curve $y = f(x)$ at $x = 3$, and write an equation for the tangent line there.

7. $y = 2x^2 - 5$
8. $y = x^2 - 6x$
9. $y = 2x^2 - 13x + 5$
10. $y = -3x^2 + 4x$
11. $y = \frac{2}{x}$
12. $y = \frac{1}{x+1}$
13. $y = \frac{x}{x+1}$
14. $y = \frac{1}{2x+1}$
15. $y = x + \frac{9}{x}$
16. $y = x - \frac{1}{x}$
17. $y = 1 + \sqrt{x}$
18. $y = \sqrt{x+1}$
19. $y = \sqrt{2x}$
20. $y = \sqrt{2x+3}$

In Exercises 21–24, find an equation for the tangent line to the curve at the given point. Then GRAPH the curve and tangent in the same viewing window.

21. $y = 4 - x^2, \quad (-1, 3)$
22. $y = (x-1)^2 + 1, \quad (1, 1)$
23. $y = \sqrt{x}, \quad (1, 1)$
24. $y = \frac{1}{x^2}, \quad (-1, 1)$

In Exercises 25–30, use the alternate derivative formula

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

from the proof of Theorem 1 to find the derivative of f at the given value of c.

25. $f(x) = x^2 - x + 1, \quad c = 1/2$
26. $f(x) = -3x^2 + 7x + 5, \quad c = 2$
27. $f(x) = \frac{1}{x+2}, \quad c = -1$
28. $f(x) = \frac{1}{(x-1)^2}, \quad c = 2$
29. $f(x) = \frac{1}{\sqrt{x}}, \quad c = 4$
30. $f(x) = \frac{1}{\sqrt{2x+13}}, \quad c = -2$

Compare the right-hand and left-hand derivatives to show that the functions in Exercises 31–34 are not differentiable at the indicated point P. Support your findings graphically.

31. $f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases} \quad P = (0, 0)$
32. $f(x) = \begin{cases} 2, & x < 1 \\ 2x, & x \geq 1 \end{cases} \quad P = (1, 2)$
33. $f(x) = \begin{cases} \sqrt{x}, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases} \quad P = (1, 1)$
34. $f(x) = \begin{cases} x, & x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases} \quad P = (1, 1)$

In Exercises 35–38, do the following:

a) Find the derivative $f'(x)$ of the given function $f(x)$.
b) GRAPH $y_1 = f(x)$, and $y_2 = f'(x)$ in the same viewing window.

Then answer these questions:

c) For what values of x, if any, is $f'(x)$ positive? Zero? Negative?
d) Over what intervals of x-values, if any, does the function $f(x)$ increase as x increases? Decrease as x increases? How is this connected with what you found in part (c)? (We say more about this connection in Chapter 4.)

35. $y = -x^2$
36. $y = -\frac{1}{x}$
37. $y = \frac{x^3}{3}$
38. $y = \frac{x^4}{4}$