Exercises 3.6

In Exercises 1–14, find dy/dx.

1. $y = \sin(3x + 1)$
2. $y = \sin(7 - 5x)$
3. $y = \cos(-x/3)$
4. $y = \cos\left(\sqrt{3}x\right)$
5. $y = \tan(2x - x^3)$
6. $y = \tan 5(2x - 1)$
7. $y = x + \sec(x^2 + \sqrt{2})$
8. $y = x\sec(3 - 8x)$
9. $y = -\csc(x^2 + 7x)$
10. $y = \sqrt{x} + \csc(1 - 2x)$
11. $y = 5\cot\left(\dfrac{2}{x}\right)$
12. $y = \cot\left(\pi - \dfrac{1}{x}\right)$
13. $y = \cos(\sin x)$
14. $y = \sec(\tan x)$

In Exercises 15–26, find dy/dx. Support your answer with the graph of $y_1 = \text{NDER}\, y$.

15. $y = (2x + 1)^5$
16. $y = (4 - 3x)^9$
17. $y = (x^2 + 1)^{-3}$
18. $y = (x + \sqrt{x})^{-2}$
19. $y = \left(1 - \dfrac{x}{7}\right)^{-7}$
20. $y = \left(\dfrac{x}{2} - 1\right)^{-10}$
21. $y = \left(\dfrac{x^2}{8} + x - \dfrac{1}{x}\right)^4$
22. $y = \left(\dfrac{x}{5} + \dfrac{1}{5x}\right)^5$
23. $y = (\csc x + \cot x)^{-1}$
24. $y = -(\sec x + \tan x)^{-1}$
25. $y = \sin^4 x + \cos^{-2} x$
26. $y = \sin^{-5} x - \cos^3 x$

Use the Chain Rule in combination with the Product and Quotient Rules to find dy/dx in Exercises 27–36.

27. $y = x^3(2x - 5)^4$
28. $y = (1 - x)(3x^2 - 5)^5$
29. $y = (4x + 3)^4(x + 1)^{-3}$
30. $y = (2x - 5)^{-1}(x^2 - 5x)^6$
31. $y = \left(\dfrac{\sin x}{1 + \cos x}\right)^2$
32. $y = \left(\dfrac{1 + \cos x}{\sin x}\right)^{-1}$
33. $y = \left(\dfrac{x}{x - 1}\right)^{-3}$
34. $y = \left(\dfrac{x}{x - 1}\right)^2 - \dfrac{4}{x - 1}$
35. $y = \sin^3 x \tan 4x$
36. $y = \cos^4 x \cot 7x$

Find dy/dx in Exercises 37–44. Support your answer with the graph of $y_1 = \text{NDER}\, y$.

37. $y = \sqrt{\sin x}$
38. $y = \sqrt{\cos x}$
39. $y = 4\sqrt{\sec x + \tan x}$
40. $y = 2\sqrt{\csc x + \cot x}$
41. $y = \dfrac{3}{\sqrt{2x + 1}}$
42. $y = \dfrac{x}{\sqrt{1 + x^2}}$
43. $y = (2x - 6)\sqrt{x + 5}$
44. $y = x\sqrt{x^2 - 2x}$

In Exercises 45–48, find ds/dt.

45. $s = \cos\left(\dfrac{\pi}{2} - 3t\right)$
46. $s = t\cos(\pi - 4t)$
47. $s = \dfrac{4}{3\pi}\sin 3t + \dfrac{4}{5\pi}\cos 5t$
48. $s = \sin\left(\dfrac{3\pi}{2}t\right) + \cos\left(\dfrac{7\pi}{4}t\right)$

In Exercises 49–52, find $dr/d\theta$.

49. $r = \tan(2 - \theta)$
50. $r = \sec 2\theta \tan 2\theta$
51. $r = \sqrt{\theta \sin\theta}$
52. $r = 2\theta\sqrt{\sec\theta}$

In Exercises 53–60, find dy/dx. You will need to use the Chain Rule two or three times in each case.

53. $y = \sin^2(3x - 2)$
54. $y = \sec^2 5x$
55. $y = (1 + \cos 2x)^2$
56. $y = (1 - \tan(x/2))^{-2}$
57. $y = \sin(\cos(2x - 5))$
58. $y = \left(1 + \cos^2 7x\right)^3$
59. $y = \cot\sqrt{2x}$
60. $y = \sqrt{\tan 5x}$

Find y'' in Exercises 61–64.

61. $y = \tan x$
62. $y = \cot x$
63. $y = \cot(3x - 1)$
64. $y = 9\tan(x/3)$

In Exercises 65–70, find the value of $(f \circ g)'$ at the given value of x.

65. $f(u) = u^5 + 1,\ u = g(x) = \sqrt{x},\ x = 1$
66. $f(u) = 1 - \dfrac{1}{u},\ u = g(x) = \dfrac{1}{1 - x},\ x = -1$
67. $f(u) = \cot\dfrac{\pi u}{10},\ u = g(x) = 5\sqrt{x},\ x = 1$
68. $f(u) = u + \dfrac{1}{\cos^2 u},\ u = g(x) = \pi x,\ x = 1/4$
69. $f(u) = \dfrac{2u}{u^2 + 1},\ u = g(x) = 10x^2 + x + 1,\ x = 0$
70. $f(u) = \left(\dfrac{u - 1}{u + 1}\right)^2,\ u = g(x) = \dfrac{1}{x^2} - 1,\ x = -1$

What happens if you can write a function as a composite in different ways? Do you get the same derivative each time? The Chain Rule says you should. Try it with the functions in Exercises 71–74.

71. Find dy/dx if $y = \cos(6x + 2)$ by writing y as a composite with
    a) $y = \cos u$ and $u = 6x + 2$
    b) $y = \cos 2u$ and $u = 3x + 1$.
72. Find dy/dx if $y = \sin(x^2 + 1)$ by writing y as a composite with
    a) $y = \sin(u + 1)$ and $u = x^2$
    b) $y = \sin u$ and $u = x^2 + 1$.
73. Find dy/dx if $y = x$ by writing y as the composite of
    a) $y = (u/5) + 7$ and $u = 5x - 35$
    b) $y = 1 + (1/u)$ and $u = 1/(x - 1)$.
74. Find dy/dx if $y = \sin(\sin(2x))$ by writing y as the composite of
    a) $y = \sin u$ and $u = \sin 2x$
    b) $y = \sin(\sin u)$ and $u = 2x$.

75. Evaluate ds/dt when $s = \cos\theta$ and $d\theta/dt = 5$ when $\theta = 3\pi/2$.

76. Evaluate dy/dt when $y = x^2 + 7x - 5$ and $dx/dt = 1/3$ when $x = 1$.

77. What is the largest value possible for the slope of the curve $y = \sin(x/2)$?

78. Write an equation for the tangent to the curve $y = \sin mx$ at the origin.

79. Find the lines that are tangent and normal to the curve $y = 2\tan(\pi x/4)$ at $x = 1$. Support your answer graphically.

80. *Orthogonal curves.* Two curves are said to cross at right angles if their tangents are perpendicular at the crossing point. The technical word for "crossing at right angles" is *orthogonal.* Show that the curves $y = \sin 2x$ and $y = -\sin(x/2)$ are orthogonal at the origin. Draw both graphs and both tangents in a square viewing window.

81. Suppose that functions f and g and their derivatives have the following values at $x = 2$ and $x = 3$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	$\frac{1}{3}$	-3
3	3	-4	2π	5

Evaluate the derivatives with respect to x of the following combinations at the given value of x.

a) $2f(x)$ at $x = 2$
b) $f(x) + g(x)$ at $x = 3$
c) $f(x) \cdot g(x)$ at $x = 3$
d) $f(x)/g(x)$ at $x = 2$
e) $f(g(x))$ at $x = 2$
f) $\sqrt{f(x)}$ at $x = 2$
g) $1/g^2(x)$ at $x = 3$
h) $\sqrt{f^2(x) + g^2(x)}$ at $x = 2$

82. Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 0$ and $x = 1$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	$-1/3$	$-8/3$

Evaluate the derivatives with respect to x of the following combinations at the given value of x:

a) $5f(x) - g(x)$, $x = 1$
b) $f(x)g^3(x)$, $x = 0$
c) $\dfrac{f(x)}{g(x)+1}$, $x = 1$
d) $f(g(x))$, $x = 0$
e) $g(f(x))$, $x = 0$
f) $(g(x) + f(x))^{-2}$, $x = 1$
g) $f(x + g(x))$, $x = 0$

83. *Running machinery too fast.* Suppose that a piston is moving straight up and down and that its position at time t sec is

$$s = A\cos(2\pi bt),$$

with A and b positive. The value of A is the amplitude of the motion, and b is the frequency (number of times the piston moves up and down each second). What effect does doubling the frequency have on the piston's velocity and acceleration? (Once you find out, you will know why machinery breaks when you run it too fast.)

84. Use parametric mode to simulate the motion of the piston in Exercise 83. Simultaneously graph the path of the piston and the velocity, both as a function of time t.

85. *Temperatures in Fairbanks, Alaska.* The equation that approximates the average temperature (°F) on day t in Fairbanks, Alaska, during a typical 365-day year is

$$y = 37\sin\left[\frac{2\pi}{365}(t - 101)\right] + 25.$$

a) Draw a complete graph of $y = f(t)$. Assume that $t = 0$ is January 1.
b) On what day is the temperature increasing the fastest?
c) About how many degrees per day is the temperature increasing when it is increasing at its fastest?

86. *Daylight hours in Fairbanks, Alaska.* The equation that approximates the number of hours of daylight on day t in Fairbanks, Alaska, during a typical 365-day year is

$$y = 8.5\sin\left(\frac{2\pi(t - 83)}{365}\right) + 12.5.$$

a) Draw a complete graph of $y = f(t)$. Assume that $t = 0$ is January 1.
b) On what day is the number of hours of daylight increasing the fastest?
c) About how many hours per day is the number of daylight hours increasing when it is increasing at its fastest?

3.7 Implicit Differentiation and Fractional Powers

Graphing Curves of the Form $F(x, y) = 0$

The equations $x^2 + y^2 = 64$, $y^2 = x$, and $y^5 + \sin xy = 0$ define relations that are *not* functions. The equation $y = x^3 + 2x - 3$ defines a function.