Quick Review 4.6

In Exercises 1 and 2, find the distance between the points A and B.

1. $A(0, 5)$, $B(7, 0)$
2. $A(0, a)$, $B(b, 0)$

In Exercises 3–6, find dy/dx.

3. $2xy + y^2 = x + y$
4. $x \sin y = 1 - xy$
5. $x^2 = \tan y$
6. $\ln(x + y) = 2x$

In Exercises 7 and 8, find a parametrization for the line segment with endpoints A and B.

7. $A(-2, 1)$, $B(4, -3)$
8. $A(0, -4)$, $B(5, 0)$

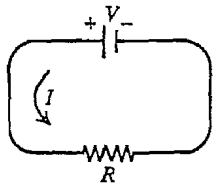
In Exercises 9 and 10, let $x = 2\cos t$, $y = 2\sin t$. Find a parameter interval that produces the indicated portion of the graph.

9. The portion in the second and third quadrants, including the points on the axes.
10. The portion in the fourth quadrant, including the points on the axes.

Section 4.6 Exercises

In Exercises 1–41, assume all variables are differentiable functions of t.

1. *Area* The radius r and area A of a circle are related by the equation $A = \pi r^2$. Write an equation that relates dA/dt to dr/dt.
2. *Surface Area* The radius r and surface area S of a sphere are related by the equation $S = 4\pi r^2$. Write an equation that relates dS/dt to dr/dt.
3. *Volume* The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$.
   (a) How is dV/dt related to dh/dt if r is constant?
   (b) How is dV/dt related to dr/dt if h is constant?
   (c) How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?
4. *Electrical Power* The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and current I (amperes) by the equation $P = RI^2$.
   (a) How is dP/dt related to dR/dt and dI/dt?
   (b) How is dR/dt related to dI/dt if P is constant?
5. *Diagonals* If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2 + y^2 + z^2}$. How is ds/dt related to dx/dt, dy/dt, and dz/dt?
6. *Area* If a and b are the lengths of two sides of a triangle, and θ the measure of the included angle, the area A of the triangle is $A = (1/2)\,ab\sin\theta$. How is dA/dt related to da/dt, db/dt, and $d\theta/dt$?
7. *Changing Voltage* The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit like the one shown here are related by the equation $V = IR$. Suppose that V is increasing at the rate of 1 volt/sec while I is decreasing at the rate of 1/3 amp/sec. Let t denote time in sec.



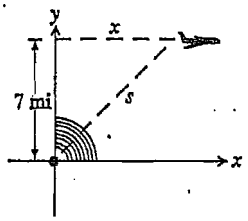
   (a) What is the value of dV/dt?
   (b) What is the value of dI/dt?
   (c) Write an equation that relates dR/dt to dV/dt and dI/dt.
   (d) **Writing to Learn** Find the rate at which R is changing when $V = 12$ volts and $I = 2$ amp. Is R increasing, or decreasing? Explain.
8. *Heating a Plate* When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/sec. At what rate is the plate's area increasing when the radius is 50 cm?
9. *Changing Dimensions In a Rectangle* The length ℓ of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $\ell = 12$ cm and $w = 5$ cm, find the rates of change of
   (a) the area, (b) the perimeter, and
   (c) the length of a diagonal of the rectangle.
   (d) **Writing to Learn** Which of these quantities are decreasing, and which are increasing? Explain.

10. *Changing Dimensions in a Rectangular Box* Suppose that the edge lengths x, y, and z of a closed rectangular box are changing at the following rates:

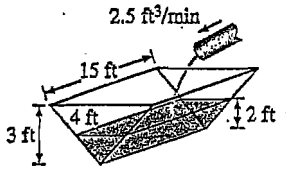
$$\frac{dx}{dt} = 1 \text{ m/sec}, \quad \frac{dy}{dt} = -2 \text{ m/sec}, \quad \frac{dz}{dt} = 1 \text{ m/sec}.$$

Find the rates at which the box's (a) volume, (b) surface area, and (c) diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ are changing at the instant when $x = 4$, $y = 3$, and $z = 2$.

11. *Air Traffic Control* An airplane is flying at an altitude of 7 mi and passes directly over a radar antenna as shown in the figure. When the plane is 10 mi from the antenna ($s = 10$), the radar detects that the distance s is changing at the rate of 300 mph. What is the speed of the airplane at that moment?



12. *Filling a Trough* A trough is 15 ft long and 4 ft across the top as shown in the figure. Its ends are isosceles triangles with height 3 ft. Water runs into the trough at the rate of 2.5 ft^3/min. How fast is the water level rising when it is 2 ft deep?

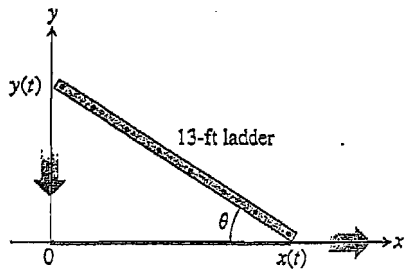


13. *Sliding Ladder* A 13-ft ladder is leaning against a house (see figure) when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

(a) How fast is the top of the ladder sliding down the wall at that moment?

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?

(c) At what rate is the angle θ between the ladder and the ground changing at that moment?



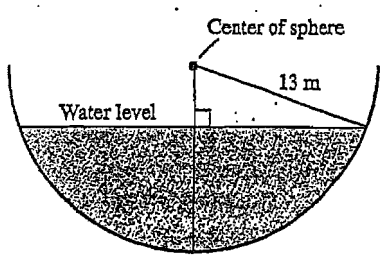
14. *Flying a Kite* Inge flies a kite at a height of 300 ft, the wind carrying the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

15. *Boring a Cylinder* The mechanics at Lincoln Automotive are reboring a 6-in. deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every 3 min. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.800 in.?

16. *Growing Sand Pile* Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high? Give your answer in cm/min.

17. *Draining Conical Reservoir* Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. (a) How fast is the water level falling when the water is 5 m deep? (b) How fast is the radius of the water's surface changing at that moment? Give your answer in cm/min.

18. *Draining Hemispherical Reservoir* Water is flowing at the rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius 13 m, shown here in profile. Answer the following questions given that the volume of water in a hemispherical bowl of radius R is $V = (\pi/3)y^2(3R - y)$ when the water is y units deep.



(a) At what rate is the water level changing when the water is 8 m deep?

(b) What is the radius r of the water's surface when the water is y m deep?

(c) At what rate is the radius r changing when the water is 8 m deep?

19. *Growing Raindrop* Suppose that a droplet of mist is a perfect sphere and that, through condensation, the droplet picks up moisture at a rate proportional to its surface area. Show that under these circumstances the droplet's radius increases at a constant rate.

20. *Inflating Balloon* A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

(a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

(b) How fast is the surface area increasing at that instant?