Exercises 4.3

In Exercises 1–8, use Newton's method to find the solutions.

1. $x^2 + x - 1 = 0$. Confirm your answers using the quadratic formula.
2. $x^3 + x - 1 = 0$. Support your answers using ZOOM-IN.
3. $x^4 + x - 3 = 0$
4. $2x - x^2 + 1 = 0$
5. $x^4 - 2x^3 - x^2 - 2x + 2 = 0$
6. $2x^4 - 4x^2 + 1 = 0$
7. $9 - \frac{3}{2}x^2 + x^3 - \frac{1}{4}x^4 = 0$
8. $x^5 - 5x^3 + 4x + 5 = 0$
9. Use Newton's method to find the positive fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_0 = 1$.
10. Use Newton's method to find the negative fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_0 = -1$.
11. Suppose your first guess in using Newton's method is lucky in the sense that x_0 is a root of $f(x) = 0$. What happens to x_1 and later approximations?
12. You plan to estimate $\pi/2$ to five decimal places by solving the equation $\cos x = 0$ by Newton's method. Does it matter what your starting value is? Explain.
13. *Oscillation.* Show that if $h > 0$, applying Newton's method to
$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$$
leads to $x_1 = -h$ if $x_0 = h$ and to $x_1 = h$ if $x_0 = -h$. Draw a picture that shows what is going on.
14. *Approximations that get worse and worse.* Apply Newton's method to $f(x) = x^{1/3}$ with $x_0 = 1$, and calculate x_1, x_2, x_3, and x_4. Find a formula for $|x_n|$. What happens to $|x_n|$ as $n \to \infty$? Draw a picture that shows what is going on.

In Exercises 15–36, show a complete graph and identify the inflection points, local maximum and minimum values, and the intervals on which the graph is rising, falling, concave up, and concave down. Indicate the number of real roots. We suggest you do exercises from the three groups in the order given.

Do Exercises 15–20 analytically, then support graphically.

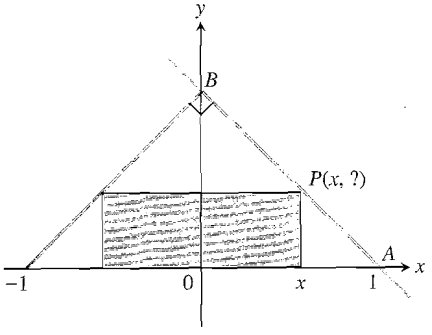
15. $y = x^3 - 3x^2 + 5x - 4$
16. $y = \frac{1}{3}x^3 - 2x^2 + 4x + 8$
17. $y = x^3 - 2x^2 - 3x + 8$
18. $y = x^3 + 10x^2 - 23x + 12$
19. $y = 12 + x - 4x^3$
20. $y = 2x^3 - x^2 - 14x - 12$

Do Exercises 21–24 graphically, then confirm analytically.

21. $y = 20 - 3x - \frac{1}{3}x^3$
22. $y = -9x^3 + 4x - 15$
23. $y = x^4 + x^2 + x + 8$
24. $y = x^4 - 8x^3 + 17x^2 - 10x - 1$

Do Exercises 25–36 using a method of your choice.

25. $y = 4x^3 - 17x^2 + 8x - 1$
26. $y = -x^3 - 3x^2 + 10x + 3$
27. $y = 10 + \frac{8}{3}x^3 - x^4$
28. $y = \frac{1}{3}x^4 + \frac{4}{3}x^3 + 2x^2 + 2x - 10$
29. $y = x^4 - 2x^2 + x + 20$
30. $y = 20 - 4x + 2x^2 + 3x^3 - \frac{9}{4}x^4$
31. $y = -x^4 - 7x^3 - 9x^2 + 7x + 12$
32. $y = x^4 - 8x^3 + 14x^2 + 8x - 5$
33. $y = -x^5 - x^4 + 11x^3 + 9x^2 - 18x + 5$
34. $y = x^5 - \frac{5}{2}x^4 - 5x^2 + 1$
35. $y = 3x^5 + \frac{15}{4}x^4 + 10x^3 + \frac{15}{2}x^2 + 15x + 1$
36. $y = \frac{1}{5}x^5 - \frac{9}{4}x^4 + \frac{19}{3}x^3 + \frac{9}{2}x^2 - 16x + 1$
37. The sum of two nonnegative numbers is 20. Find the numbers if the sum of their squares is to be as large as possible.
38. Show that among all rectangles with an 8-ft perimeter, the one with the largest area is a square.
39. You are planning to make an open rectangular box from an 8-in. × 15-in. piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way?
40. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose?
41. The figure below shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.
    a) Express the y-coordinate of P in terms of x. (You might start by writing an equation for the line AB.)



    b) Express the area of the rectangle in terms of x.
    c) What is the largest area the rectangle can have?