What do we do when our calculus tells us that the best production level is a value of $x$ that isn't an integer, as it did in Example 4 when it said that $x = 2 + \sqrt{21}/3$ thousand units would be the production level for maximum profit? The answer is to use the nearest "convenient" integer. For $x = 2 + \sqrt{21}/3$ thousand, we might use 3528, or 3530, or 3500, depending perhaps on a "convenience factor" such as whether the items will be shipped in boxes of 12, 10, or 100.

## Exercises 4.4

In Exercises 1–26, show a complete graph, and identify the inflection points, local maximum and minimum values, and the intervals on which the graph is rising, falling, concave up, and concave down. We suggest that you do exercises from the three groups in the order given.

Do Exercises 1–6 analytically, then support graphically.

**1.** $y = \dfrac{x^2 - 1}{x}$ **2.** $y = \dfrac{x^2 + 4}{2x}$

**3.** $y = \dfrac{x^4 + 1}{x^2}$ **4.** $y = \dfrac{x^3 + 1}{x^2}$

**5.** $y = \dfrac{x}{x^2 - 4}$ **6.** $y = \dfrac{x - 1}{x^3 - 2x^2}$

Do Exercises 7–12 graphically, then confirm analytically.

**7.** $y = \dfrac{1}{x^2 - 1}$ **8.** $y = \dfrac{x^2}{x^2 - 1}$

**9.** $y = -\dfrac{x^2 - 2}{x^2 - 1}$ **10.** $y = \dfrac{x^2 - 4}{x^2 - 2}$

**11.** $y = \dfrac{x^2 - 4}{x - 1}$ **12.** $y = -\dfrac{x^2 - 4}{x + 1}$

Do Exercises 13–26 using a method of your choice.

**13.** $y = x^2 - x + \dfrac{1}{x + 2}$ **14.** $y = x + \dfrac{4}{x^2}$

**15.** $y = \dfrac{x + 1}{x^2 + 1}$ **16.** $y = \dfrac{x^2 + 1}{x^3 - 4x}$

**17.** $y = \dfrac{2x^3 + 3x^2 - 2x + 1}{x^2 + 2x}$

**18.** $y = \dfrac{3x^3 - 7x^2 - 7x - 1}{x^2 - 3x}$

**19.** $y = \dfrac{2x^3 - 4x^2 + 3}{x - 2}$

**20.** $y = \dfrac{-x^3 - 3x^2 + x + 5}{x + 3}$

**21.** $y = \dfrac{x^4 - 3x^2 + 4x + 1}{x^2 + x - 2}$

**22.** $y = \dfrac{2x^4 + 3x^3 + x^2 + x - 3}{x^2 + x}$

**23.** $y = \dfrac{2x^5 - 18x^3 + 2}{x^2 - 9}$

**24.** $y = \dfrac{x^5 + 3x^4 - 11x^3 - 3x^2 + 9x - 4}{x^2 + 3x - 10}$

**25.** $y = \dfrac{8}{x^2 + 4}$ (Agnesi's witch)

**26.** $y = \dfrac{4x}{x^2 + 4}$ (Newton's serpentine)

**27.** What is the smallest perimeter possible for a rectangle whose area is 16 in.$^2$?

**28.** You are planning to make an open rectangular box with a square base that will hold a volume of 50 ft$^3$. What are the dimensions of the box with minimum surface area?

**29.** A 216-m$^2$ rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

**30.** *The lightest steel holding tank.* Your iron works has contracted to design and build a 500-ft$^3$, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding half-inch-thick stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible. What dimensions do you tell the shop to use? Explain.

**31.** *Catching rainwater.* An 1125-ft$^3$ open-top rectangular tank with a square base $x$ ft on a side and $y$ ft deep is to be built with its top flush with the ground to catch runoff water. The costs associated with the tank involve not only the material from which the tank is made but also an excavation charge proportional to the product $xy$. If the total cost is

$$c = 5(x^2 + 4xy) + 10xy,$$

what values of $x$ and $y$ will minimize it?

**32.** You are designing a poster to contain 50 in.$^2$ of printing with margins of 4 in. each at top and bottom and 2 in. at each side. What overall dimensions will minimize the amount of paper used?