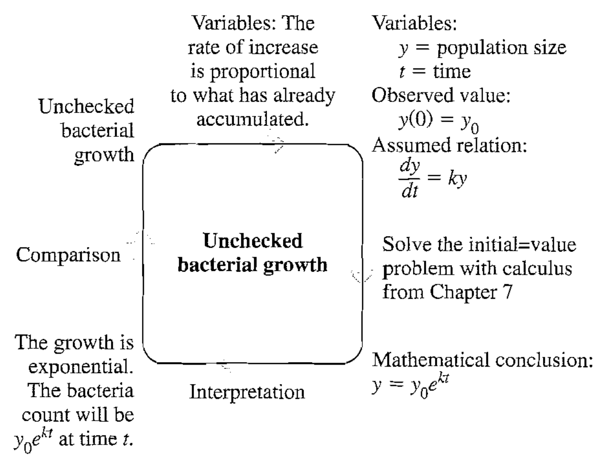
Models in Biology

You may have noticed that we haven't mentioned models in biology yet. The reason is that most mathematical models of life processes use either exponential functions or logarithms, functions whose exact derivatives are not found until Chapter 7. Typical of the models that we will study there is the model for unchecked bacterial growth. The basic assumption is that at any time t, the rate dy/dt at which the population is changing is proportional to the number $y(t)$ of bacteria present. If the population's original size is y_0, this leads to the initial-value problem

$$\text{Differential equation:} \quad \frac{dy}{dt} = ky$$

$$\text{Initial condition:} \quad y = y_0 \quad \text{when} \quad t = 0.$$

As you will see, the solution turns out to be $y = y_0 e^{kt}$, so the modeling cycle looks like this:



This model is one of the really good models that we talked about earlier, because it applies to so many of the phenomena that we want to forecast and understand: cell growth, heat transfer, radioactive decay, the flow of electrical current, and the accumulation of capital by compound interest, to mention only a few. We will see how all of this works by the time we are through with Chapter 7.

Exercises 4.7

Find the general antiderivatives of the functions in Exercises 1–18. Do as many as you can without writing anything down (except the answer). Then support your answers with a graphing utility.

1. **a)** $2x$ **b)** x^2 **c)** $x^2 - 2x + 1$

2. **a)** $6x$ **b)** x^5 **c)** $x^5 - 6x + 3$

3. **a)** $-3x^{-4}$ **b)** x^{-4} **c)** $x^{-4} + 2x + 3$

4. **a)** $2x^{-3}$ **b)** $\frac{x^{-3}}{2} + x^2$ **c)** $-x^{-3} + x - 1$

5. **a)** $\frac{1}{x^2}$ **b)** $\frac{5}{x^2}$ **c)** $2 - \frac{5}{x^2}$

6. **a)** $-\frac{2}{x^3}$ **b)** $\frac{1}{2x^3}$ **c)** $x^3 - \frac{1}{x^3}$

7. **a)** $\frac{3}{2}\sqrt{x}$ **b)** $4\sqrt{x}$ **c)** $x^2 - 4\sqrt{x}$

8. **a)** $\frac{4}{3}\sqrt[3]{x}$ **b)** $\frac{1}{3\sqrt[3]{x}}$ **c)** $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

9. **a)** $\frac{2}{3}x^{-1/3}$ **b)** $\frac{1}{3}x^{-2/3}$ **c)** $-\frac{1}{3}x^{-4/3}$

10. **a)** $\frac{1}{2}x^{-1/2}$ **b)** $-\frac{1}{2}x^{-3/2}$ **c)** $-\frac{3}{2}x^{-5/2}$

11. **a)** $-\sin 3x$ **b)** $3\sin x$ **c)** $3\sin x - \sin 3x$

12. **a)** $\pi \cos \pi x$ **b)** $\frac{\pi}{2}\cos\frac{\pi x}{2}$ **c)** $\cos\frac{\pi x}{2}$

13. **a)** $\sec^2 x$ **b)** $5\sec^2 5x$ **c)** $\sec^2 5x$

14. **a)** $\csc^2 x$ **b)** $7\csc^2 7x$ **c)** $\csc^2 7x$

15. **a)** $\sec x \tan x$ **b)** $2\sec 2x \tan 2x$ **c)** $4\sec 2x \tan 2x$

16. **a)** $\csc x \cot x$ **b)** $8\csc 4x \cot 4x$ **c)** $\csc 4x \cot 4x$

17. $(\sin x - \cos x)^2$ (*Hint:* $2\sin x\cos x = \sin 2x$)

18. $(1 + 2\cos x)^2$ (*Hint:* $2\cos^2 x = 1 + \cos 2x$)

19. Suppose that $1 - \sqrt{x}$ is an antiderivative of $f(x)$ and that $x + 2$ is an antiderivative of $g(x)$. Find the *general* antiderivatives of the following functions.

a) $f(x)$ **b)** $g(x)$ **c)** $-f(x)$

d) $-g(x)$ **e)** $f(x) + g(x)$ **f)** $3f(x) - 2g(x)$

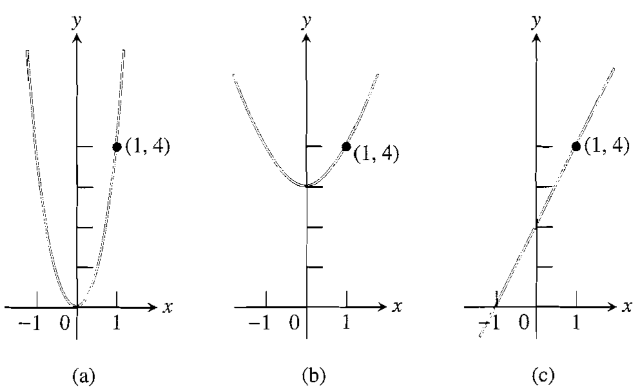
g) $x + f(x)$ **h)** $g(x) - 4$

20. Repeat Exercise 19, assuming that e^x is an antiderivative of $f(x)$ and that $x \sin x$ is an antiderivative of $g(x)$.

21. Which of the following graphs shows the solution of the initial-value problem

$$\frac{dy}{dx} = 2x, \qquad y = 4 \quad \text{when} \quad x = 1?$$

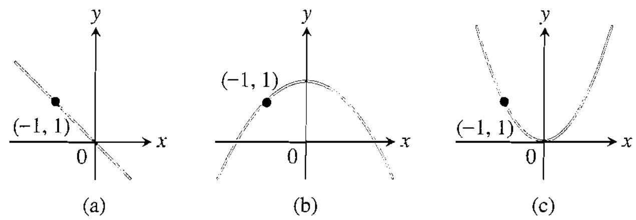
Give reasons for your answer.



22. Which of the following graphs shows the solution of the initial-value problem

$$\frac{dy}{dx} = -x, \qquad y = 1 \quad \text{when} \quad x = -1?$$

Give reasons for your answer.



Solve the initial-value problems in Exercises 23–34 for y as a function of x. If possible, support your answers with a graphing utility.

23. $\frac{dy}{dx} = 2x - 7,\ y = 0$ when $x = 2$

24. $\frac{dy}{dx} = 10 - x,\ y = -1$ when $x = 0$

25. $\frac{dy}{dx} = x^2 + 1,\ y = 1$ when $x = 0$

26. $\frac{dy}{dx} = x^2 + \sqrt{x},\ y = 1$ when $x = 1$

27. $\frac{dy}{dx} = -5/x^2,\ x > 0;\ y = 3$ when $x = 5$

28. $\frac{dy}{dx} = \frac{1}{x^2} + x,\ x > 0;\ y = 1$ when $x = 2$

29. $\frac{dy}{dx} = 3x^2 + 2x + 1,\ y = 0$ when $x = 1$

30. $\frac{dy}{dx} = 9x^2 - 4x + 5,\ y = 0$ when $x = -1$

31. $\frac{dy}{dx} = 1 + \cos x,\ y = 4$ when $x = 0$

32. $\frac{dy}{dx} = \cos x + \sin x,\ y = 1$ when $x = \pi$

33. $\frac{d^2y}{dx^2} = 2 - 6x,\ y = 1$ and $\frac{dy}{dx} = 4$ when $x = 0$

34. $\frac{d^3y}{dx^3} = 6;\ y = 5,\ \frac{dy}{dx} = 0,$ and $\frac{d^2y}{dx^2} = -8$ when $x = 0$

Exercises 35 and 36 give the velocity and initial position of a body moving along a coordinate line. Find the body's position at time t. Simulate the motion with a grapher in parametric mode.

35. $v = 9.8t,\ s = 10,$ when $t = 0$

36. $v = \sin t,\ s = 0,$ when $t = 0$

Exercises 37 and 38 give the acceleration, initial velocity, and initial position of a body moving along a coordinate line. Find the body's position at time t. Simulate the motion with a grapher in parametric mode.

37. $a = 32,\ v = 20,$ and $s = 0$ when $t = 0$

38. $a = \sin t,\ v = -1,$ and $s = 1$ when $t = 0$

39. Find the curve in the xy-plane that passes through the point (9, 4) and whose slope at each point is $3\sqrt{x}$.

40. **a)** Find a function $y = f(x)$ with the following properties:

i) $\dfrac{d^2y}{dx^2} = 6x$.

ii) Its graph in the xy-plane passes through the point (0, 1) and has a horizontal tangent there.

b) How many functions like this are there? How do you know?

41. *Revenue from marginal revenue.* Suppose that the marginal revenue when x thousand units are sold is

$$\frac{dr}{dx} = 3x^2 - 6x + 12$$

dollars per unit. Find the revenue function $r(x)$ given that there is no revenue if no units are sold.

42. *Cost from marginal cost.* Suppose that the marginal cost of manufacturing an item when x thousand items are produced is

$$\frac{dc}{dx} = 3x^2 - 12x + 15$$

dollars per item. Find the cost function $c(x)$ if $c(0) = 400$.

43. On the moon, the acceleration of gravity is 1.6 m/sec^2. If a rock is dropped into a crevasse, how fast will it be going just before it hits bottom 30 sec later?

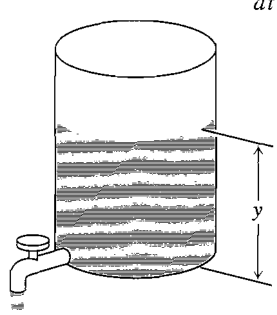
44. A rocket lifts off the surface of Earth with a constant acceleration of 20 m/sec^2. How fast will the rocket be going 1 min later?

45. With approximately what velocity do you enter the water if you dive from a 10-m platform? (Use $g = 9.8$ m/sec^2.)

46. The acceleration of gravity near the surface of Mars is 3.72 m/sec^2. If a rock is blasted straight up from the surface with an initial velocity of 93 m/sec (about 208 mi/h), how high does it go? (*Hint:* When is the velocity zero?)

47. *How long will it take a tank to drain?* If we open a valve to drain the water from a cylindrical tank, the water will flow fast when the tank is full but slow down as the tank drains. It turns out that the rate at which the water level drops is proportional to the square root of the water's depth. In the notation of the diagram this means that

$$\frac{dy}{dt} = -k\sqrt{y}. \qquad (4)$$



The value of k depends on the acceleration of gravity and the cross-sectional areas of the tank and drain hole. Equation (4) has a negative sign because y decreases with time. To solve Eq. (4), rewrite it as

$$\frac{1}{\sqrt{y}}\frac{dy}{dt} = -k, \qquad (5)$$

and carry out the following steps.

a) Find the general antiderivative of each side of Eq. (5).

b) Set the antiderivatives in part (a) equal, and combine their arbitrary constants into a single arbitrary constant. (Nothing is achieved by having two when one will do.) This will give an equation that relates y directly to t.

48. *Continuation of Exercise 47.* (a) Suppose t is measured in minutes and $k = 1/10$. Find y as a function of t if $y = 9$ ft when $t = 0$. (b) How long does it take the tank to drain if the water is 9 ft deep to start with?

49. *Two ferris wheels (continued from Exploration 2).* Let $y = D(t)$ be the distance between Renee and Sherrie.

a) Show that $y = D(t)$ is a periodic function, and determine its period.

b) Draw a complete graph of $y = D(t)$.

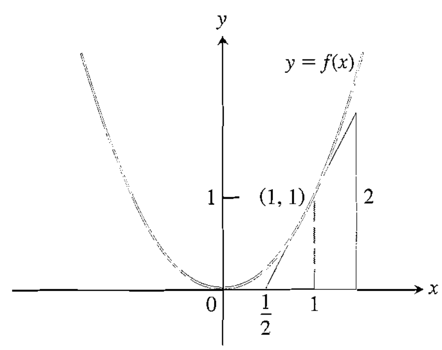
c) Find the maximum and minimum distances between Renee and Sherrie and the first time each value occurs.

d) Explain how you would try to confirm the results in part (c) analytically. Can you do it?

50. Repeat Exercise 49 assuming that Renee and Sherrie start at (0, 0) and (15, 0), respectively, when $t = 0$.

51. The graph below is that of a function $y = f(x)$ that solves one of the following initial-value problems. Which one? How do you know?

a) $dy/dx = 2x$, $y(1) = 0$, **b)** $dy/dx = x^2$, $y(1) = 1$,
c) $dy/dx = 2x + 2$, $y(1) = 1$, **d)** $dy/dx = 2x$, $y(1) = 1$.



52. Give a convincing argument that

$$y = \frac{2}{3}(x^4 + x^2 + 3)^{3/2} + 2$$

is not the solution for the initial-value problem

$$dy/dx = (x^4 + x^2 + 3)^{1/2}(4x^3 + 2x),\ y(0) = 2.$$

Use the technique described in Example 10 to sketch solution curves of the differential equations in Exercises 53–56. Then solve the differential equations, and graph the solutions to support your work. Also support graphically with SLOPEFLD.

53. $\frac{dy}{dx} = 2x$

54. $\frac{dy}{dx} = -2x + 2$

55. $\frac{dy}{dx} = 1 - 3x^2$

56. $\frac{dy}{dx} = x^2$

Use the technique described in Example 10 to sketch the solution curve of each initial-value problem in Exercises 57–60. Support graphically using SLOPEFLD.

57. $\frac{dy}{dx} = \sqrt{1 + x^4}, \quad y = 1 \quad \text{when } x = 0$

58. $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}, \quad -1 < x < 1; y = 0 \quad \text{when } x = 0$

59. $\frac{dy}{dx} = \frac{x}{x^2 + 1}, y = 0 \quad \text{when } x = 0$

60. $\frac{dy}{dx} = \frac{1}{x^2 + 1} - 1, \quad y = 1 \quad \text{when } x = 0$

61. Use SLOPEFLD to approximate the solution curves to $dy/dt = 0.001y(100 - y)$ in a [0, 100] by [0, 100] viewing window. Then sketch the particular solution $y = f(t)$ that satisfies $f(0) = 10$.

Chapter 4 Review Questions

1. What does it mean for a function $y = f(x)$ to have an absolute or local maximum or minimum value?
2. How do you find the local and absolute maximum and minimum values of a function $y = f(x)$?
3. What are the hypotheses and conclusion of Rolle's Theorem? How does the theorem sometimes help you to tell how many solutions an equation has in a given interval?
4. What are the hypotheses and conclusion of the Mean Value Theorem? What physical interpretation does the theorem sometimes have? Give an example.
5. This chapter gives three important corollaries of the Mean Value Theorem. State each one and describe how it is used.
6. How do you test a function to find out where its graph is concave up or concave down? What is an inflection point? What physical significance do inflection points sometimes have?
7. State the First Derivative Test for Local Extreme Values.
8. State the Second Derivative Test for Local Extreme Values.
9. List the steps that you would take to confirm a computer-generated graph of a function. How does calculus tell you the shape of the graph between plotted points? Give an example.
10. Give a general description of the class of polynomial functions. Indicate the possible number of real zeros and the possible number of local extrema.
11. Describe Newton's method for solving equations. Give an example. What is the theory behind the method? What are some of the things to watch out for when you use the method?
12. Describe how you would solve max-min problems. Illustrate with an example.
13. What guidance do you get from calculus about finding production levels that maximize profit? That minimize average manufacturing cost?
14. Describe how you would sketch the graph of a rational function.
15. Indicate a reasonable way to describe the behavior of a periodic function.
16. Describe how you would use a graphing utility to draw the graph of $f(x) = \log_a x$.
17. Describe how you would solve related rate problems. Illustrate with an example.
18. What is an antiderivative of a function $y = f(x)$? When a function has an antiderivative, how do we find its general antiderivative? Illustrate with an example.
19. What general rules can you call on to help find antiderivatives? Show, by example, how they are used.
20. What is an initial-value problem? How do you solve one? Illustrate with an example.

Chapter 4 Practice Exercises

1. Show that $y = x/(x + 1)$ increases on every interval in its domain.
2. Show that $y = \sin^2 t - 3t$ decreases on every interval in its domain.
3. Show that $y = x^3 + 2x$ has no maximum or minimum values.
4. Does $f(x) = x^3 + 2x + \tan x$ have any local maximum or minimum values?

5. If $f'(x) \leq 2$ for all x, what is the most f can increase on the interval $0 \leq x \leq 6$?

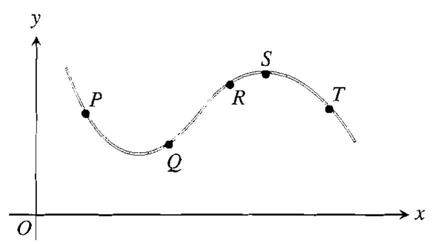
6. Show that the equation $x^4 + 2x^2 - 2 = 0$ has exactly one solution on the interval $0 \leq x \leq 1$.

In Exercises 7 and 8, suppose that the first derivative of $y = f(x)$ is as given. At what points, if any, does the graph of f have a local maximum, local minimum, or point of inflection?

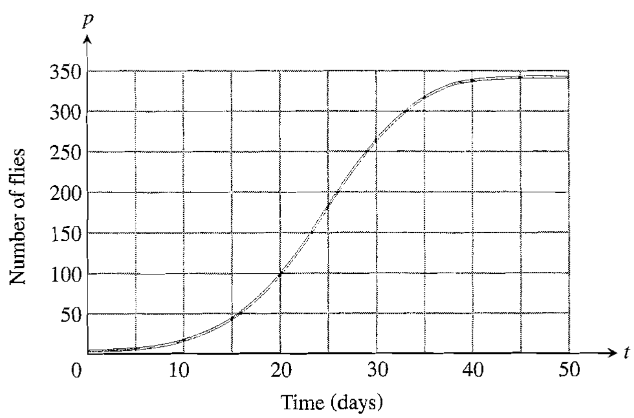
7. $y' = 6(x+1)(x-2)^2$

8. $y' = 6x(x+1)(x-2)$

9. At which of the five points on the graph of $y = f(x)$ shown here (a) are y' and y'' both negative? (b) is y' negative and y'' positive?



10. Here is the graph of the fruit fly population again. On approximately what day did the population's growth rate change from increasing to decreasing?



(a)

Q	Slope of $PQ = \Delta p/\Delta t$ (flies/day)
(45, 340)	(340 − 150)/(45 − 23) ≈ 8.64
(40, 330)	(330 − 150)/(40 − 23) ≈ 10.59
(35, 310)	(310 − 150)/(35 − 23) ≈ 13.33
(30, 265)	(265 − 150)/(30 − 23) ≈ 16.43

(b)

In Exercises 11–30, show a complete graph and identify the inflection points, local maximum and minimum values, and the intervals on which the graph is rising, falling, concave up, and concave down. We suggest that you do exercises from the three groups in the order given.

Do Exercises 11–14 analytically, then support graphically.

11. $y = -x^3 - 3x^2 - 4x - 2$

12. $y = x^3 - 9x^2 - 21x - 11$

13. $y = \sqrt[3]{x-2}$

14. $y = \sqrt[4]{1-x}$

Do Exercises 15 and 16 graphically, then confirm analytically.

15. $y = 1 + x - x^2 - x^4$

16. $y = \frac{2}{3}x^3 + 5x + 20$

Do Exercises 17–30 using a method of your choice.

17. $y = -\frac{8}{3}x^3 + 4x^2 - 2x - 12$

18. $y = -x^4 + 4x^3 - 4x^2 + x + 20$

19. $y = x^4 - \frac{8}{3}x^3 - \frac{x^2}{2} + 1$

20. $y = 4x^5 + 5x^4 + \frac{20}{3}x^3 + 4$

21. $y = -x^5 + \frac{7}{3}x^3 + 5x^2 + 4x + 2$

22. $y = \frac{1}{5}x^5 + \frac{3}{2}x^4 + \frac{5}{3}x^3 - 6x^2 + 3x + 1$

23. $y = \dfrac{5 - 4x + 4x^2 - x^3}{x - 2}$

24. $y = \dfrac{3x^3 - 5x^2 - 11x - 11}{x^2 - 2x - 3}$

25. $y = \log_3 |x|$

26. $y = e^{x-1} - x$

27. $y = x \log(x-2)$

28. $y = \sin 3x + \cos 4x$

29. $y = \sqrt[4]{x - x^2}$

30. $y = \sinh(x+2)$

31. Use Newton's method to find where the curve $y = -x^3 + 3x + 4$ crosses the x-axis. Support graphically.

32. Use Newton's method to solve the equation $\sec x = 4$ on the interval $0 \leq x \leq \pi/2$. Support graphically.

33. Use Newton's method to solve the equation $2\cos x - \sqrt{1+x} = 0$.

34. Find the approximate values of r_1 through r_4 in the factorization

$$8x^4 - 14x^3 - 9x^2 + 11x - 1 = 8(x-r_1)(x-r_2)(x-r_3)(x-r_4).$$

35. *Estimating reciprocals without division.* Newton's method in Section 4.3 can be used to estimate the reciprocal of a positive number a without ever dividing by a, by taking $f(x) = 1/x - a$. For example, if $a = 3$, the function involved is $f(x) = 1/x - 3$.
   a) Graph $y = 1/x - 3$. Where does the graph cross the x-axis?
   b) Show that the recursion formula in Newton's method in this case is

$$x_{n+1} = x_n(2 - 3x_n),$$

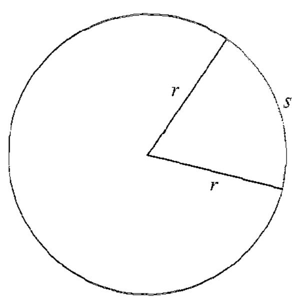
so indeed there is no division.

36. Show that the equation $x^3 + x - 1 = 0$ has exactly one solution, and use Newton's method to find it to three decimal places.

37. Find the maximum and minimum values of $f(x) = 10 + 20x - 11x^2 - 8x^3 - x^4$ on $-6 \leq x \leq 1$, and say where they are assumed.

38. Find the maximum and minimum of $f(x) = \sqrt{x} + \cos x$ on $0 \leq x \leq 11$, and say where they are assumed.

39. If the perimeter of the circular sector shown here is 100 ft, what values of r and s will give the sector the greatest area?



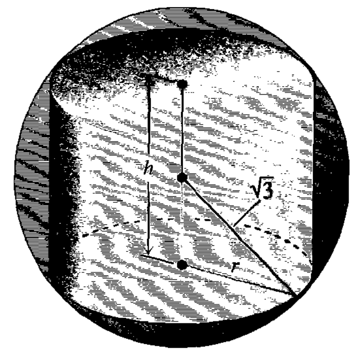
Area: $A = \frac{1}{2}rs$
Perimeter: $p = s + 2r$

40. An isosceles triangle has its vertex at the origin and its base parallel to the x-axis with the vertices above the axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.

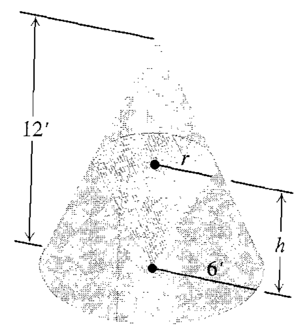
41. Find the dimensions of the largest open storage bin with a square base and vertical sides that can be made from 108 ft^2 of sheet steel. (Neglect the thickness of the steel, and assume that there is no waste.)

42. You are to design an open-top rectangular stainless-steel vat. It is to have a square base and a volume of 32 ft^3, to be welded from quarter-inch plate, and weigh no more than necessary. What dimensions do you recommend?

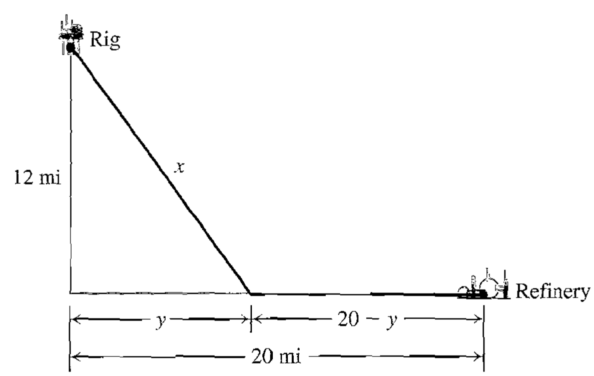
43. Find the height and radius of the largest right circular cylinder that can be put in a sphere of radius $\sqrt{3}$. (See following diagram.)



44. The figure shows two right circular cones, one upside down inside the other. The two bases are parallel, and the vertex of the smaller cone lies at the center of the larger cone's base. What values of r and h will give the smaller cone the largest possible volume?



45. A drilling rig 12 mi off shore is to be connected by a pipe to a refinery on shore, 20 mi down the coast from the rig. If underwater pipe costs \$40,000 per mile and land-based pipe costs \$30,000 per mile, what values of x and y give the least expensive connection?



46. An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the

two ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?

47. Your company can manufacture x hundred grade A tires and y hundred grade B tires a day, where $0 \le x \le 4$ and

$$y = \frac{40 - 10x}{5 - x}.$$

Your profit on Grade A tires is twice your profit on grade B tires. What is the most profitable number of each kind of tire to make?

48. The positions of two particles on the s-axis are $s_1 = \sin t$ and $s_2 = \sin(t + \pi/3)$.

a) What is the farthest apart the particles ever get?

b) When, if ever, do the particles collide?

49. An open-top rectangular box is constructed from a 10- by 16-in. piece of cardboard by cutting squares of equal side length from the corners and folding up the sides. Find analytically the dimensions of the box of largest volume and the maximum volume. Support your results graphically.

50. a) Repeat Exercise 67 in Section 4.3 for a 22- by 34-in. piece of cardboard. In part (f), replace 1120 in.^{3} by 400 in.^{3}

b) Repeat Exercise 68 in Section 4.3 for a 12- by 20-in. piece of cardboard. There are two possibilities.

Describe the motion of the particle moving on the x-axis with distance from the origin given by the functions in Exercises 51 and 52. Simulate the motion with a grapher in parametric mode.

51. $s(t) = t^3 + t^2 - 6t + 5$

52. $s(t) = 3 + 4t - 3t^2 - t^3$

53. The radius of a circle is changing at the rate of $-2/\pi$ m/sec. At what rate is the circle's area changing when $r = 10$ m?

54. The coordinates of a particle moving in the metric xy-plane are differentiable functions of time t with $dx/dt = -1$ m/sec and $dy/dt = -5$ m/sec. How fast is the particle approaching the origin as it passes through the point (5, 12)?

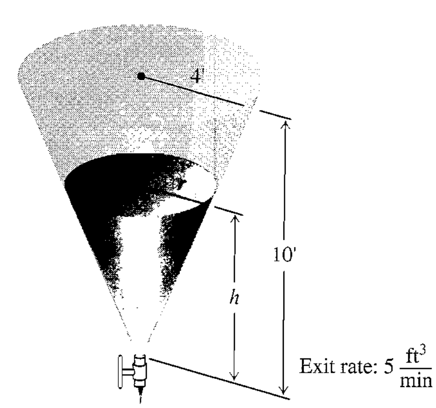
55. The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the edges changing at that instant?

56. A point moves smoothly along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the constant rate of 11 units per second. Find dx/dt when $x = 3$.

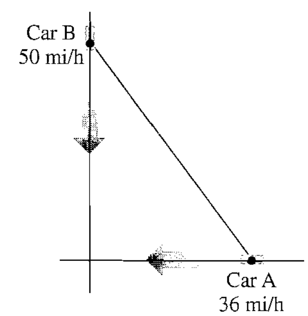
57. Water drains from the conical tank shown in the following diagram at the rate of 5 ft^3/min.

a) What is the relation between the variables h and r?

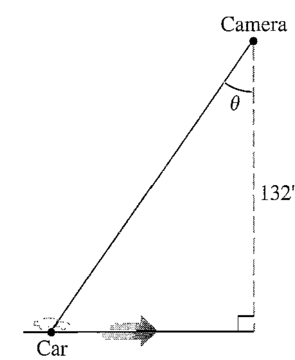
b) How fast is the water level dropping when $h = 6$ ft?



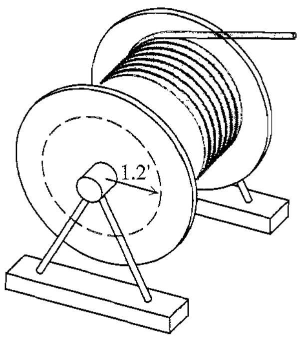
58. Two cars are approaching an intersection along straight highways that cross at right angles, car A moving at 36 mi/h and car B at 50 mi/h. At what rate is the straight-line distance between the cars changing when car A is 5 mi and car B is 12 mi from the intersection? At what rate is the distance changing at any time?



59. You are videotaping a race from a stand 132 ft from the track, following a car that is traveling at 180 mi/h (264 ft/sec). How fast will your camera angle θ be changing when the car is right in front of you? A half-second later?



60. As telephone cable is pulled from a large spool to be strung from the telephone poles along a street, it unwinds from the spool in layers of constant radius. If the truck pulling the cable moves at a steady 6 ft/sec (a touch over 4 mi/h), use the equation $s = r\theta$ to find how fast (in radians per second) the spool is turning when the layer of radius 1.2 ft is being unwound?



61. The formula $F(x) = 3x + C$ gives a different function for each value of C. All of these functions, however, have the same derivative with respect to x, namely, $F'(x) = 3$. Are these the only differentiable functions whose derivative is 3? Could there be any others? Explain.

62. Show that

$$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{d}{dx}\left(-\frac{1}{x+1}\right)$$

even though

$$\frac{x}{x+1} \neq -\frac{1}{x+1}.$$

Doesn't this contradict Corollary 3 of the Mean Value Theorem? Explain.

63. Find the general antiderivatives of the following functions.

a) 0 **b)** 1 **c)** x
d) x^2 **e)** x^{10} **f)** x^{-2}
g) x^{-5} **h)** $x^{5/2}$ **i)** $x^{4/3}$
j) $x^{3/4}$ **k)** $x^{1/2}$ **l)** $x^{-1/2}$
m) $x^{-3/7}$ **n)** $x^{-7/3}$

64. Find the general antiderivatives of the following functions.

a) $\sin x$ **b)** $\cos x$ **c)** $\sec x \tan x$
d) $-\csc^2 x$ **e)** $\sec^2 x$ **f)** $-\csc x \cot x$

Find the general antiderivatives of the functions in Exercises 65–80. Then support your answer with a graphing utility.

65. $3x^2 + 5x - 7$

66. $\frac{1}{x^2} + x + 1$

67. $\sqrt{x} + \frac{1}{\sqrt{x}}$

68. $\sqrt[3]{x} + \sqrt[4]{x}$

69. $3\cos 5x$

70. $8\sin(x/2)$

71. $3\sec^2 3x$

72. $4\csc^2 2x$

73. $\frac{1}{2} - \cos x$

74. $3x^5 + 16\cos 8x$

75. $\sec\frac{x}{3}\tan\frac{x}{3} + 5$

76. $1 - \csc x \cot x$

77. $\tan^2 x$ (*Hint:* $\tan^2 x = \sec^2 x - 1$.)

78. $\cot^2 x$ (*Hint:* $\cot^2 x = \csc^2 x - 1$.)

79. $2\sin^2 x$ (*Hint:* $2\sin^2 x = 1 - \cos 2x$.)

80. $\sin^2 x - \cos^2 x$ (*Hint:* $\cos^2 x - \sin^2 x = \cos 2x$.)

Solve the initial-value problems in Exercises 81–86. If possible, support your answer with a graphing utility.

81. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}$, $y = 1$ when $x = 0$

82. $\frac{dy}{dx} = 4x^3 - 21x^2 + 14x - 7$, $y = 1$ when $x = 1$

83. $\frac{dy}{dx} = \frac{x^2+1}{x^2}$, $y = -1$ when $x = 1$

84. $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$, $y = 1$ when $x = 1$

85. $\frac{d^2y}{dx^2} = -\sin x$, $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

86. $\frac{d^2y}{dx^2} = \cos x$, $y = -1$ and $\frac{dy}{dx} = 0$ when $x = 0$

87. Does any function $y = f(x)$ satisfy all of the following conditions? If so, what is it? If not, why not?
a) $d^2y/dx^2 = 0$ for all x
b) $dy/dx = 1$ when $x = 0$
c) $y = 0$ when $x = 0$

88. Find an equation for the curve in the xy-plane that passes through the point $(1, -1)$ if its slope at x is always $3x^2 + 2$.

89. You sling a shovelful of dirt up from the bottom of a 17-ft hole with an initial velocity of 32 ft/sec. Is that enough speed to get the dirt out of the hole, or had you better duck?

90. The acceleration of a particle moving along a coordinate line is $d^2s/dt^2 = 2 + 6t$ m/sec^2. At $t = 0$, the velocity is 4 m/sec. Find the velocity as a function of t. Then find how far the particle moves during the first second of its trip, from $t = 0$ to $t = 1$.

Sketch the solution curves of the initial-value problems in Exercises 91 and 92. Support graphically with the toolbox program SLOPEFLD.

91. $\frac{dy}{dx} = 4 - \sqrt{x^2 + 3}$, $y = 0$ when $x = 0$

92. $\frac{dy}{dx} = \sqrt{x^2 + 1} - 1$, $y = 1$ when $x = 0$