Exercises 5.1

For each function $y = f(x)$ in Exercises 1–6, consider the area of the region between the graph of $y = f(x)$ and the x-axis from $x = a$ to $x = b$.

a) Make sketches illustrating the RAM for LRAM_5, RRAM_5, and MRAM_5 showing the five approximating rectangles.

b) Write out by hand LRAM_5, RRAM_5, and MRAM_5, and compute each sum.

1. $y = 6 - x^2$ from $x = 0$ to $x = 2$
2. $y = x^2 + 2$ from $x = -3$ to $x = 2$
3. $f(x) = x + 1$ from $x = 0$ to $x = 5$
4. $f(x) = 5 - x$ from $x = 0$ to $x = 5$
5. $f(x) = 2x^2$ from $x = 0$ to $x = 5$
6. $f(x) = x^2 + 2$ from $x = 1$ to $x = 6$

For each function f in Exercises 7–14, estimate the area of the region between the graph of f and the x-axis from $x = a$ to $x = b$ using the RAM for $n = 10$, 100, and 1000. Do all three RAMs: $\text{LRAM}_n f$, $\text{RRAM}_n f$, and $\text{MRAM}_n f$. First verify that each function is nonnegative on the specified interval $[a, b]$.

7. $f(x) = x^2 - x + 3$ from $x = 0$ to $x = 3$
8. $f(x) = 2x^2 - 5x + 6$ from $x = -1$ to $x = 4$
9. $f(x) = 2x^3 + 3$ from $x = 0$ to $x = 5$
10. $f(x) = x^3 + x^2 + 2x + 3$ from $x = -1$ to $x = 3$
11. $f(x) = \sin x$ from $x = 0$ to $x = \pi$
12. $f(x) = \cos x$ from $x = 0$ to $x = \dfrac{\pi}{2}$
13. $f(x) = e^{-x^2}$ from $x = -5$ to $x = 5$
14. $f(x) = 2 + \dfrac{\sin x}{x}$ from $x = -3$ to $x = 4$
15. Make a conjecture about the *exact* area of the region described in Exercises 3, 5, 7, 9, and 11.
16. Make a conjecture about the *exact* area of the region described in Exercises 4, 6, 8, 10, and 12.
17. Confirm the values in Table 5.1.
18. Confirm the values in Table 5.2.

Write the sums in Exercises 19–28 without sigma notation. Then evaluate them.

19. $\displaystyle\sum_{k=1}^{4} \frac{1}{k}$
20. $\displaystyle\sum_{k=1}^{4} \frac{12}{k}$
21. $\displaystyle\sum_{k=1}^{3} (k+2)$
22. $\displaystyle\sum_{k=1}^{5} (2k-1)$
23. $\displaystyle\sum_{k=0}^{4} \frac{k}{4}$
24. $\displaystyle\sum_{k=-2}^{2} 3k$
25. $\displaystyle\sum_{k=1}^{4} \cos k\pi$
26. $\displaystyle\sum_{k=1}^{3} \sin \frac{\pi}{k}$
27. $\displaystyle\sum_{k=1}^{4} (-1)^k$
28. $\displaystyle\sum_{k=1}^{4} (-1)^{k+1}$
29. Which of the following express $1 + 2 + 4 + 8 + 16 + 32$ in sigma notation?

    a) $\displaystyle\sum_{k=1}^{6} 2^{k-1}$ b) $\displaystyle\sum_{k=0}^{5} 2^k$ c) $\displaystyle\sum_{k=-1}^{4} 2^{k+1}$
30. Which formula is not equivalent to the others?

    a) $\displaystyle\sum_{k=-1}^{1} \frac{(-1)^k}{k+2}$ b) $\displaystyle\sum_{k=0}^{2} \frac{(-1)^k}{k+1}$

    c) $\displaystyle\sum_{k=1}^{3} \frac{(-1)^k}{k}$ d) $\displaystyle\sum_{k=2}^{4} \frac{(-1)^{k-1}}{k-1}$

Express the sums in Exercises 31–36 in sigma notation.

31. $1 + 2 + 3 + 4 + 5 + 6$
32. $1 + 4 + 9 + 16$
33. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16}$
34. $1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}$
35. $\dfrac{1}{5} - \dfrac{2}{5} + \dfrac{3}{5} - \dfrac{4}{5} + \dfrac{5}{5}$
36. $-\dfrac{1}{5} + \dfrac{2}{5} - \dfrac{3}{5} + \dfrac{4}{5} - \dfrac{5}{5}$

Use algebra and the formulas in Eqs. (1)–(3) to evaluate the sums in Exercises 37–44.

37. $\displaystyle\sum_{k=1}^{10} k$
38. $\displaystyle\sum_{k=1}^{7} 2k$
39. $\displaystyle\sum_{k=1}^{6} -k^2$
40. $\displaystyle\sum_{k=1}^{6} (k^2 + 5)$
41. $\displaystyle\sum_{k=1}^{5} k(k-5)$
42. $\displaystyle\sum_{k=1}^{7} (2k - 8)$
43. $\displaystyle\sum_{k=1}^{100} k^3 - \sum_{k=1}^{99} k^3$
44. $\displaystyle\left(\sum_{k=1}^{7} k\right)^2 - \sum_{k=1}^{7} k^3$