from its third derivative, we would have to find the values of three constants of integration, and so on. In each case, the values of the constants are determined by the problem's initial conditions. Each time we integrate, we need an initial condition to tell us the value of C.

EXPLORATION 3

Solving an Initial Value Problem

We have seen that a family of antiderivatives of f that are part of the indefinite integral $\int f$ can be represented by $F(x) = \int_a^x f(t)\,dt$ where a represents an arbitrary real number. Suppose we have the initial value problem:

$$\text{Differential equation: } \frac{dy}{dx} = (0.6)2^x,$$

$$\text{Initial condition: } \quad y = -1 \quad \text{when} \quad x = 0$$

and we want to find the value of a, if any, so that $F(x) = \int_a^x f(t)\,dt$ is the solution of the problem (that is, $F(x)$ includes the constant of integration so that F satisfies the initial condition and F' satisfies the given differential equation). Then

$$-1 = F(0) = \int_a^0 f(t)\,dt = -\int_0^a f(t)\,dt, \qquad \text{or} \qquad 1 = \int_0^a f(t)\,dt.$$

1. For $f(x) = (0.6)2^x$, GRAPH $y_1 = \int_0^x f(t)\,dt = \text{NINT}\,(f(t), 0, x)$. Use ZOOM-IN or SOLVE to find a where $\int_0^a f(t)\,dt = \text{NINT}\,(f(t), 0, a) = 1$.
2. In part 1, you should find $a = 1.108$, accurate to three decimal places. The solution to the initial value problem, then, is
$$y = \int_{1.108}^x (0.6)2^t\,dt.$$
In a later section, we will find the representation $(0.6)2^x/\ln 2 - 1.866$ to be the solution. For now, GRAPH $y_1 = \text{NINT}\,((0.6)2^t, 1.108, x)$ and $y_2 = (0.6)2^x/\ln 2 - 1.866$ for support that they are the same solution of the differential equation. (What would you expect the y-intercept of each to be? Why?)

Exercises 5.5

Evaluate each integral in Exercises 1–10. Confirm by differentiating your answer and comparing the result with the integrand.

1. $\int x^3\,dx$
2. $\int 7\,dx$
3. $\int (x+1)\,dx$
4. $\int (6-6x)\,dx$
5. $\int 3\sqrt{x}\,dx$
6. $\int \frac{4}{x^2}\,dx$
7. $\int x^{-1/3}\,dx$
8. $\int (1-4x^{-3})\,dx$
9. $\int (5x^2+2x)\,dx$
10. $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right)\,dx$

Evaluate each integral in Exercises 11–20. Support by comparing the graph of NDER of your answer with the graph of the integrand.

11. $\int (2x^3-5x+7)\,dx$
12. $\int (1-x^2-3x^5)\,dx$
13. $\int 2\cos x\,dx$
14. $\int 5\sin\theta\,d\theta$

15. $\int \sin \frac{x}{3}\, dx$

16. $\int 3 \cos 5x\, dx$

17. $\int 3 \csc^2 x\, dx$

18. $\int \frac{\sec^2 x}{3}\, dx$

19. $\int \frac{\csc x \cot x}{2}\, dx$

20. $\int \frac{2}{5} \sec x \tan x\, dx$

Evaluate each integral in Exercises 21–30. Support by comparing the graph of the antiderivative you found and $y = \text{NINT}\,(f(t), 0, x)$ or by using SLOPEFLD.

21. $\int (4 \sec x \tan x - 2 \sec^2 x)\, dx$

22. $\int \frac{1}{2}(\csc^2 x - \csc x \cot x)\, dx$

23. $\int (\sin 2x - \csc^2 x)\, dx$

24. $\int (2 \cos 2x - 3 \sin 3x)\, dx$

25. $\int 4 \sin^2 y\, dy$

26. $\int \frac{\cos^2 x}{7}\, dx$

27. $\int \sin x \cos x\, dx$ (*Hint:* $2 \sin x \cos x = \sin 2x$)

28. $\int (1 - \cos^2 t)\, dt$

29. $\int (1 + \tan^2 \theta)\, d\theta$ (*Hint:* $1 + \tan^2 \theta = \sec^2 \theta$)

30. $\int \frac{1 + \cot^2 x}{2}\, dx$

In Exercises 31 and 32, write the equation of the continuous extension of the function in the specified closed interval. Graph $y = \int_0^x f(t)\, dt$ for the continuous extension of f.

31. $f(x) = \frac{1 - \cos x}{x^2}, \quad -10 \le x \le 10$

32. $f(x) = \left(1 + \frac{1}{x}\right)^x, \quad 0 \le x \le 5$

Show that the integral formulas in Exercises 33–36 are correct by showing that the derivative of the right-hand side is the integrand in the integral on the left-hand side. (In Section 5.6 we will see where formulas like these come from.)

33. $\int (7x - 2)^3\, dx = \frac{(7x - 2)^4}{28} + C$

34. $\int \sec^2 5x\, dx = \frac{\tan 5x}{5} + C$

35. $\int \frac{1}{(x + 1)^2}\, dx = -\frac{1}{x + 1} + C$

36. $\int \frac{1}{(x + 1)^2}\, dx = \frac{x}{x + 1} + C$

37. Right or wrong? Say which for each formula.

a) $\int x \sin x\, dx = \frac{x^2}{2} \sin x + C$

b) $\int x \sin x\, dx = -x \cos x + C$

c) $\int x \sin x\, dx = -x \cos x + \sin x + C$

38. Right or wrong? Say which for each formula.

a) $\int (2x + 1)^2\, dx = \frac{(2x + 1)^3}{3} + C$

b) $\int 3(2x + 1)^2\, dx = (2x + 1)^3 + C$

c) $\int 6(2x + 1)^2\, dx = (2x + 1)^3 + C$

Solve each initial value problem in Exercises 39–46 for y as a function of x. GRAPH the solution $y = \text{NINT} f$ if analytic solutions are unavailable.

39. Differential equation: $\frac{dy}{dx} = 3\sqrt{x}$

Initial condition: $y = 4$ when $x = 9$

40. Differential equation: $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

Initial condition: $y = 0$ when $x = 4$

41. Differential equation: $\frac{dy}{dx} = 2^x$

Initial condition: $y = 2$ when $x = 0$

42. Differential equation: $\frac{dy}{dx} = \frac{1 - \cos x}{x}$

Initial condition: $y = 0.5$ when $x = 0$

43. Differential equation: $\frac{d^2y}{dx^2} = 0$

Initial conditions: $\frac{dy}{dx} = 2$ and $y = 0$ when $x = 0$

44. Differential equation: $\frac{d^2y}{dx^2} = \frac{2}{x^3}$

Initial conditions: $\frac{dy}{dx} = 1$ and $y = 1$ when $x = 1$

45. Differential equation: $\frac{d^2y}{dx^2} = \frac{3x}{8}$

Initial conditions: $\frac{dy}{dx} = 3$ and $y = 4$ when $x = 4$

46. Differential equation: $\frac{d^3y}{dx^3} = 6$

Initial conditions: $\frac{d^2y}{dx^2} = -8$, $\frac{dy}{dx} = 0$, and $y = 5$ when $x = 0$