is usually brutal. For both of the above reasons, it becomes vital, we feel, that you learn to use the calculator and become much better equipped from this course to make it work for you.

EXPLORATION 5

Integrals in the Real World

Just for fun, here for you to evaluate are three definite integrals based on Exploration 4. First identify how each integral differs from the integral in Exploration 4. Then evaluate each, using pencil and paper or a graphing utility. It's your choice.

1. $\int_{-1}^{1} \sqrt{x^3 + 1}\, dx$
2. $\int_{2.1}^{5.6} 3x^2\sqrt{x^3 + 1}\, dx$
3. $\int_{2.1}^{5.6} \sqrt{x^3 + 1}\, dx$

(With a NINT tolerance of 0.00001, we found the values to be 1.95, 1542.87, and 27.39 on our graphing calculator. All three took us a total of 80 seconds including keystroking and some shortcuts. By now, we hope that you know some calculator shortcuts and have been able to share them with others.)

Exercises 5.6

Evaluate each indefinite integral in Exercises 1–10 by using the given substitution to reduce the integral to a standard form. Support by graphing the antiderivative and $y = \text{NINT}\,(f(t), 0, x)$ in the same viewing window.

1. $\int \sin 3x\, dx,\ u = 3x$
2. $\int x \sin(2x^2)\, dx,\ u = 2x^2$
3. $\int \sec 2x \tan 2x\, dx,\ u = 2x$
4. $\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt,\ u = 1 - \cos\frac{t}{2}$
5. $\int 28(7x-2)^3\, dx,\ u = 7x - 2$
6. $\int 4x^3(x^4-1)^2\, dx,\ u = x^4 - 1$
7. $\int \frac{9r^2\, dr}{\sqrt{1-r^3}},\ u = 1 - r^3$
8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\, dy,\ u = y^4 + 4y^2 + 1$
9. $\int \csc^2 2\theta \cot 2\theta\, d\theta$
   a) Using $u = \cot 2\theta$ b) Using $u = \csc 2\theta$
10. $\int \frac{dx}{\sqrt{5x}}$
   a) Using $u = 5x$ b) Using $u = \sqrt{5x}$

Evaluate each definite integral in Exercises 11–18. Support with a NINT computation.

11. $\int_0^{1/2} \frac{dx}{(2x+1)^3}$
12. $\int_0^1 \sqrt{5x+4}\, dx$
13. $\int_0^{\pi/6} \frac{\sin 2x}{\cos^2 2x}\, dx$
14. $\int_{\pi/6}^{\pi/2} \sin^2\theta \cos\theta\, d\theta$
15. $\int_{-1}^{1} x\sqrt{1-x^2}\, dx$
16. $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$
17. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{(2+\sin x)^2}\, dx$
18. $\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

Evaluate each indefinite integral in Exercises 19–28. Support by comparing the graph of NDER of your answer with the graph of the integrand.

19. $\int \frac{dx}{(1-x)^2}$

20. $\int \frac{4y}{\sqrt{2y^2+1}}\, dy$

21. $\int \sec^2 (x+2)\, dx$

22. $\int \sec^2 \left(\frac{x}{4}\right) dx$

23. $\int 8r(r^2-1)^{1/3}\, dr$

24. $\int x^4(7-x^5)^3\, dx$

25. $\int \sec\left(\theta + \frac{\pi}{2}\right) \tan\left(\theta + \frac{\pi}{2}\right) d\theta$

26. $\int \sqrt{\tan x}\, \sec^2 x\, dx$

27. $\int \frac{6x^3}{\sqrt[4]{1+x^4}}\, dx$

28. $\int (s^3+2s^2-5s+6)^2(3s^2+4s-5)\, ds$

Evaluate each definite integral in Exercises 29–47 by an analytic method of your choice.

29. a) $\int_0^3 \sqrt{y+1}\, dy$ **b)** $\int_{-1}^0 \sqrt{y+1}\, dy$

30. a) $\int_0^1 r\sqrt{1-r^2}\, dr$ **b)** $\int_{-1}^1 r\sqrt{1-r^2}\, dr$

31. a) $\int_0^{\pi/4} \tan x \sec^2 x\, dx$ **b)** $\int_{-\pi/4}^0 \tan x \sec^2 x\, dx$

32. a) $\int_0^1 x^3(1+x^4)^3\, dx$ **b)** $\int_{-1}^1 x^3(1+x^4)^3\, dx$

33. a) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\, dx$ **b)** $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\, dx$

34. a) $\int_{-1}^1 \frac{x}{(1+x^2)^2}\, dx$ **b)** $\int_0^1 \frac{x}{(1+x^2)^2}\, dx$

35. a) $\int_0^{\sqrt{7}} x(x^2+1)^{1/3}\, dx$ **b)** $\int_{-\sqrt{7}}^0 x(x^2+1)^{1/3}\, dx$

36. a) $\int_0^{\pi} 3\cos^2 x \sin x\, dx$ **b)** $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx$

37. a) $\int_0^{\pi/6} (1-\cos 3x) \sin 3x\, dx$

b) $\int_{\pi/6}^{\pi/3} (1-\cos 3x) \sin 3x\, dx$

38. a) $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$ **b)** $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$

39. a) $\int_0^{2\pi} \frac{\cos x}{\sqrt{2+\sin x}}\, dx$ **b)** $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{2+\sin x}}\, dx$

40. a) $\int_{-\pi/2}^0 \frac{\sin x}{(3+\cos x)^2}\, dx$ **b)** $\int_0^{\pi/2} \frac{\sin x}{(3+\cos x)^2}\, dx$

41. $\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\, dt$

42. $\int_0^3 t\sqrt{1+t}\, dt$

43. $\int_0^{\pi/2} \cos^3 2x \sin 2x\, dx$

44. $\int_{-\pi/4}^{\pi/4} \tan^2 x \sec^2 x\, dx$

45. $\int_0^{\pi} \frac{8\sin t}{\sqrt{5-4\cos t}}\, dt$

46. $\int_0^{\pi/4} (1-\sin 2t)^{3/2} \cos 2t\, dt$

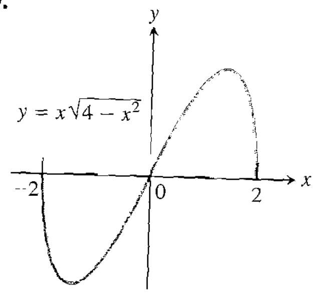
47. $\int_0^1 15x^2\sqrt{5x^3+4}\, dx$

48. Do analytically and support with a NINT computation:

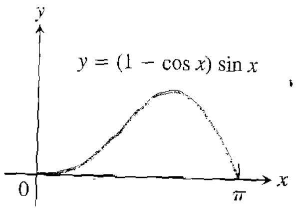
$$\int_0^1 (y^3+6y^2-12y+5)(y^2+4y-4)\, dy.$$

Find the total areas of each shaded region in Exercises 49 and 50.

49.



50.



Solve each initial value problem in Exercises 51–54.

51. $\frac{ds}{dt} = 24t(3t^2-1)^3$, $s = 0$ when $t = 0$

52. $\frac{dy}{dx} = 4x(x^2+8)^{-1/3}$, $y = 0$ when $x = 0$

53. $\frac{ds}{dt} = 6\sin(t+\pi)$, $s = 0$ when $t = 0$

54. $\frac{d^2s}{dt^2} = -4\sin\left(2t - \frac{\pi}{2}\right)$, $\frac{ds}{dt} = 100$ and $s = 0$ when $t = 0$