

Exercises 8.1

Answers to odd-numbered problems begin on page A-31

In Problems 1–4 state the domain of the given function.

1. $f(x) = \ln(x + 1)$ 2. $f(x) = \ln(3 - x)$
 3. $f(x) = \ln|x^2 - 1|$ 4. $f(x) = \ln(x^2 - 1)$

In Problems 5–10 use Theorem 8.1 to determine whether f and g are the same functions.

5. $f(x) = \ln x^6$ 6. $f(x) = \ln x^{1/3}$
 $g(x) = 6 \ln x$ $g(x) = \frac{1}{3} \ln x$
 7. $f(x) = \ln x(x^4 + 3)$ 8. $f(x) = \ln(2x + 7)$
 $g(x) = \ln x + \ln(x^4 + 3)$ $g(x) = \ln 2x + \ln 7$
 9. $f(x) = \frac{\ln(x^2 + 9)}{\ln(x^2 + 1)}$
 $g(x) = \ln(x^2 + 9) - \ln(x^2 + 1)$
 10. $f(x) = \ln(1/x)$
 $g(x) = -\ln x$

In Problems 11–34 find the derivative of the given function.

11. $y = 10 \ln x$ 12. $y = \ln 10 x$
 13. $y = \ln x^{1/2}$ 14. $y = (\ln x)^{1/2}$
 15. $y = \ln(x^4 + 3x^2 + 1)$ 16. $y = \ln(x^2 + 1)^{20}$
 17. $y = x^2 \ln x^3$ 18. $y = x - \ln|5x + 1|$
 19. $y = \frac{\ln x}{x}$ 20. $y = x(\ln x)^2$
 21. $y = \ln \frac{x}{x + 1}$ 22. $y = \frac{\ln 4x}{\ln 2x}$
 23. $y = -\ln|\cos x|$
 24. $y = \ln(x + \sqrt{x^2 - 1})$
 25. $y = \frac{1}{\ln x}$ 26. $y = \ln \frac{1}{x}$
 27. $f(x) = \ln(x \ln x)$ 28. $g(x) = \sqrt{\ln \sqrt{x}}$
 29. $f(x) = \ln(\ln(\ln x))$ 30. $w(\theta) = \theta \sin(\ln 5\theta)$
 31. $H(t) = \ln t^2(3t^2 + 6)$
 32. $G(t) = \ln \sqrt{5t + 1}(t^3 + 4)^6$
 33. $f(x) = \ln \frac{(x + 1)(x + 2)}{x + 3}$
 34. $f(x) = \ln \sqrt{\frac{(3x + 2)^5}{x^4 + 7}}$

In Problems 35–40 use implicit differentiation to find dy/dx .

35. $y^2 = \ln xy$ 36. $y = \ln(x + y)$
 37. $x + y^2 = \ln \frac{x}{y}$ 38. $y = \ln xy^2$
 39. $xy = \ln(x^2 + y^2)$ 40. $x^2 + y^2 = \ln(x + y)^2$
 41. Find an equation of the tangent line to the graph of $y = \ln x$ at $x = 1$.
 42. Find the slope of the tangent to the graph of $y = (\ln|x|)^2$ at $x = 1$.
 43. Find an equation of the tangent line to the graph of $y = \ln(x^2 - 3)$ at $x = 2$.
 44. Find the slope of the tangent to the graph of y' at the point where the slope of the tangent to the graph of $y = \ln x^2$ is 4.
 45. Determine the point on the graph of $y = \ln 2x$ at which the tangent line is perpendicular to $x + 4y = 1$.
 46. If $y = \ln x$, find $d^n y/dx^n$.

In Problems 47–50 sketch the graph of the given function.

47. $y = -\ln x$ 48. $y = 2 + \ln x$
 49. $y = \ln(x - 2)$ 50. $y = \ln|x + 1|$
 51. Answer the following questions about the graph of $f(x) = \ln(x^2 + 1)$: Intercepts? Symmetry? Asymptotes? Relative extrema? Concavity? Sketch the graph of f .
 52. Compare the graphs of $y = \ln x^2$ and $y = 2 \ln x$.
 53. For $x > 0$ verify that both $y = x^{-1/2}$ and $y = x^{-1/2} \ln x$ satisfy the equation $4x^2 d^2y/dx^2 + 8x dy/dx + y = 0$.
 54. For $x > 0$ verify that $y = C_1 x^{-1} \cos(\sqrt{2} \ln x) + C_2 x^{-1} \sin(\sqrt{2} \ln x)$, where C_1 and C_2 are constants, satisfies the equation $x^2 y'' + 3xy' + 3y = 0$.

Calculator Problems

In Problems 55 and 56 show graphically that the given equation possesses only one real root. Use Newton's Method to approximate the root to three decimal places.

55. $\ln x = 2$ 56. $x + \ln x - 3 = 0$

Miscellaneous Problems

57. Use (i) and (iii) of Theorem 8.1 to prove (ii) of Theorem 8.1. (Hint: $\ln a/b = \ln(a \cdot 1/b)$.)