

Remark

The numbers e and π are transcendental as well as irrational numbers. A transcendental number is one that is *not* a root of a polynomial equation with integer coefficients. For example, $\sqrt{2}$ is irrational but is not transcendental since it is a root of the polynomial equation $x^2 - 2 = 0$. The number e was proved to be transcendental by the French mathematician Charles Hermite (1822–1901) in 1873, whereas π was proved to be transcendental nine years later by the German mathematician Ferdinand Lindemann. The latter proof showed conclusively that “squaring a circle” with a rule and a compass was impossible.

Exercises 8.2

Answers to odd-numbered problems begin on page A-32

In Problems 1–30 find the derivative of the given function.

1. $y = e^{-x}$

2. $y = e^{2x+3}$

3. $y = e^{\sqrt{x}}$

4. $y = e^{\cos 10x}$

5. $y = \frac{e^{-2x}}{x}$

6. $y = x^3 e^{4x}$

7. $y = \sqrt{1 + e^{-5x}}$

8. $y = \frac{1}{(e^{2x} - e^{-2x})^2}$

9. $y = \ln(x^4 + e^{x^2})$

10. $y = \ln \sqrt{e^x + e^{-x}}$

11. $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

12. $y = \frac{e^{7x}}{e^{-x}}$

13. $y = e^{\ln x}$

14. $y = \ln e^{x^2}$

15. $y = e^{3x} \ln(x^2 + 1)$

16. $y = \frac{\ln x}{e^x}$

17. $y = e^{\frac{x+2}{x-2}}$

18. $y = \ln \left| \frac{1 + e^{2x}}{1 - e^{2x}} \right|$

19. $y = e^{e^{x^2}}$

20. $y = (e^3)^{x-1}$

21. $y = e^{2x} e^{3x} e^{4x}$

22. $y = e^x + e^{x+e^x}$

23. $F(t) = e^{t^{1/3}} + (e^t)^{1/3}$

24. $g(t) = e^{-t} \tan e^t$

25. $f(x) = (2x + 1)^3 e^{-(1-x)^4}$

26. $f(x) = e^{x\sqrt{x^2+1}}$

27. $f(x) = \frac{xe^x}{x + e^x}$

28. $f(x) = xe^{2x} \ln x$

29. $f(x) = \tan^{-1} e^{2x}$

30. $f(x) = x \sec e^{-x}$

In Problems 31–36 use implicit differentiation to find dy/dx .

31. $y = e^{x+y}$

32. $\ln y = x + e^y$

33. $y = \cos e^{xy}$

34. $y = e^{(x+y)^2}$

35. $x + y^2 = e^{x/y}$

36. $e^x + e^y = y$

In Problems 37–40 find the indicated derivative.

37. $y = e^{-4x}; \frac{d^3y}{dx^3}$

38. $y = \sin e^{2x}; \frac{d^2y}{dx^2}$

39. $y = \ln(e^x + 1); \frac{d^2y}{dx^2}$

40. $y = xe^x; \frac{d^4y}{dx^4}$

41. Find an equation of the tangent line to the graph of $y = e^x$ at $x = 1$.

42. Find an equation of the tangent line to the graph of $y = \ln(e^x + 1)$ at $x = 0$.

43. Find the slope of the normal line to the graph of $y = (x - 1)e^{-x}$ at $x = 0$.

44. Find the point on the graph of $y = e^x$ at which the tangent line is parallel to $3x - y = 7$.

In Problems 45–48 sketch the graph of the given function.

45. $y = -e^x$

46. $y = 1 + e^{-x}$

47. $y = 2 - e^{-x}$

48. $y = e^x + e^{-x}$

In Problems 49–54 find the relative extrema of each function. Sketch the graph.

49. $y = xe^{-x}$

50. $y = \frac{e^x}{x}$

51. $y = e^{-x^2}$

52. $y = e^{(x-2)^2}$

53. $y = x \ln x$