Combining Integrals with Formulas from Geometry

Regardless of the method that we choose for finding the area of a region, we have found that viewing a diagram, when possible, is often a desirable early step. Other steps, however, need not be limited to the use of a graphing utility or analytic methods. Indeed, we can bring to bear on a problem *any* mathematics we know.

EXAMPLE 5

If we view the region in Examples 3 and 4 as the area between the curve $y = \sqrt{x}, 0 \le x \le 4$, and the x-axis, *minus* the area of a triangle with base 2 and height 2, then

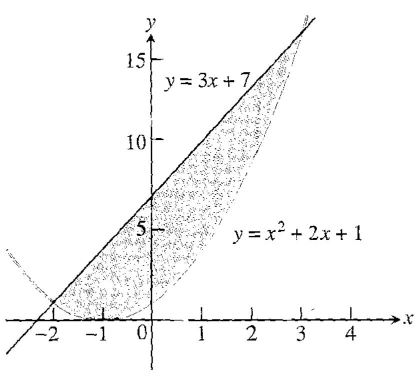
$$\text{Area} = \int_0^4 \sqrt{x}\, dx - \frac{1}{2}(2)(2) = \frac{2}{3}x^{3/2}\Big]_0^4 - 2$$
$$= \frac{2}{3}(8) - 0 - 2 = \frac{10}{3}.$$

Moral of Examples 3–5 We view the curves first. It is sometimes easier to find the area between them by integrating with respect to y instead of x. The picture may also suggest how familiar geometry formulas could be used to simplify our work.

Exercises 6.1

Find the areas of the shaded regions in Exercises 1–6 analytically. Support with a grapher.

1.



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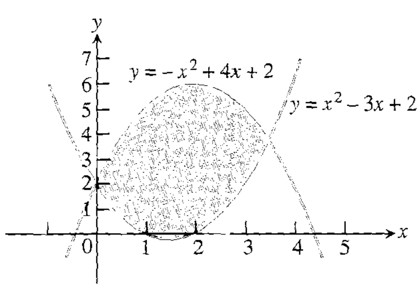

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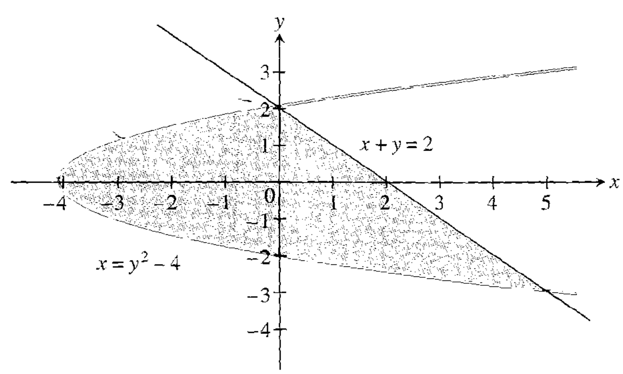

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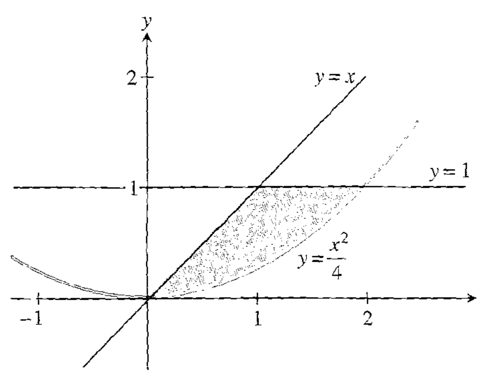

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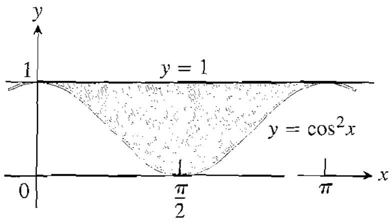

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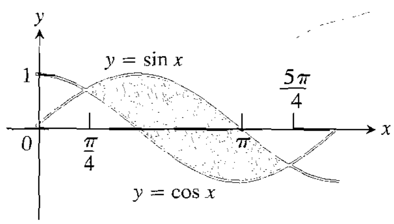


Find the area of the regions enclosed by the lines and curves in Exercises 7–22 any way you can. If you find the area analytically, support with a grapher. If you find the area graphically, confirm analytically if possible.

7. The curve $y = x^2 - 2$ and the line $y = 2$

8. The x-axis and the curve $y = 2x - x^2$

9. The curve $y^2 = x$ and the line $x = 4$

10. The curve $y = 2x - x^2$ and the line $y = -3$

11. The curve $y = x^2$ and the line $y = x$

12. The curve $x = 3y - y^2$ and the line $x + y = 3$

13. The line $y = 2x$ and the curve $y = x^3 + 2x^2 - 3x + 1$

14. Below the line $y = 3 - 2x$ and above the curve $y = 2\cos 2x$ in the first quadrant.

15. The curve $y = x^2 - 2x$ and the line $y = x$

16. The curve $x = 10 - y^2$ and the line $x = 1$

17. Above the line $y = 2x$ and below the curve $y = e^{-x^2}$ in the first quadrant

18. The curve $y = e^x$ and the lines $y = -x$ and $x = 2$

19. The line $y = x$ and the curve $y = 2 - (x - 2)^2$

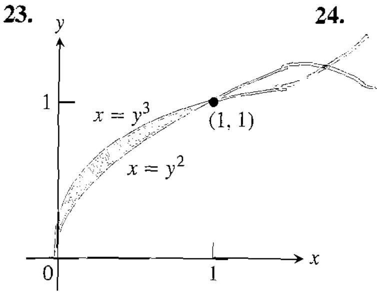
20. The curves $y = 7 - 2x^2$ and $y = x^2 + 4$

21. The line $y = x$ and the curve $y = x^3 - 2x^2 - 3x + 1$ (*Hint*: There are two regions.)

22. The curve $y = 2 - x^2$ and $y = 2\cos 2x$ (*Hint*: There are two regions.)

Find the areas of the shaded regions in Exercises 23–26 graphically. Confirm analytically.

23.



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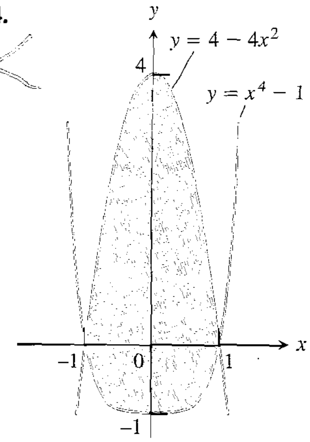

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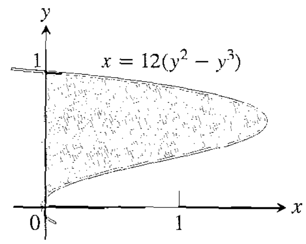
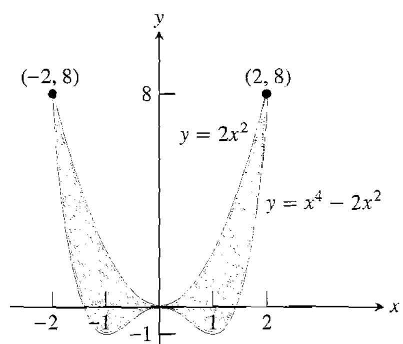


26.



NOT TO SCALE

27. Find the area of the "triangular" region bounded by the y-axis and the curves $y = \sin x$ and $y = \cos x$ in the first quadrant.

28. Find the area of the region between the curve $y = 3 - x^2$ and the line $y = -1$ by integrating with respect to **(a)** x; **(b)** y.

29. The area of the region between the curve $y = x^2$ and the line $y = 4$ is divided into two equal portions by the line $y = c$.

a) Find c by integrating with respect to y. (This puts c into the limits of integration.)

b) Find c by integrating with respect to x. (This puts c into the integrand as well.)