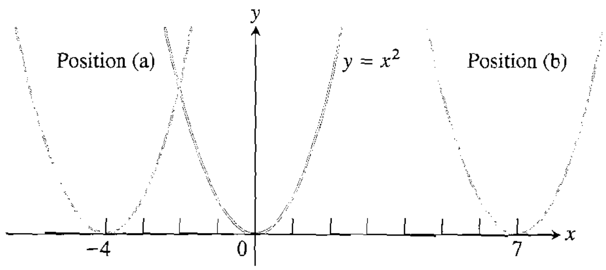
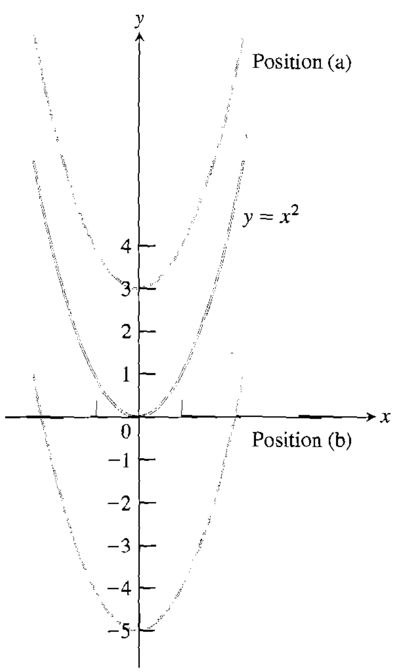
Exercises 1.4

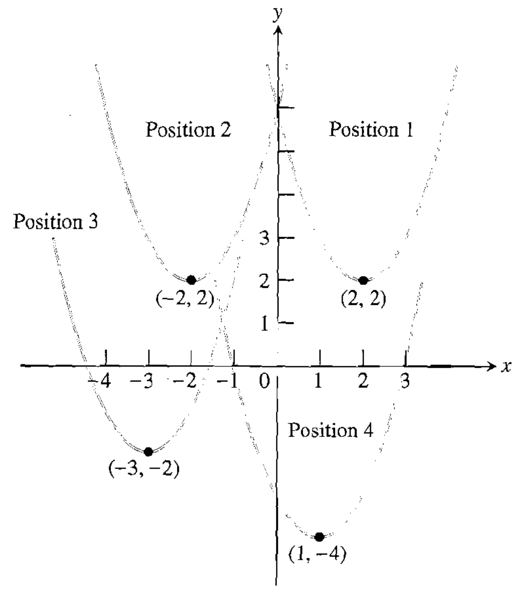
1. The graph of $y = x^2$ is shifted to two new positions. Write equations for the new graphs.



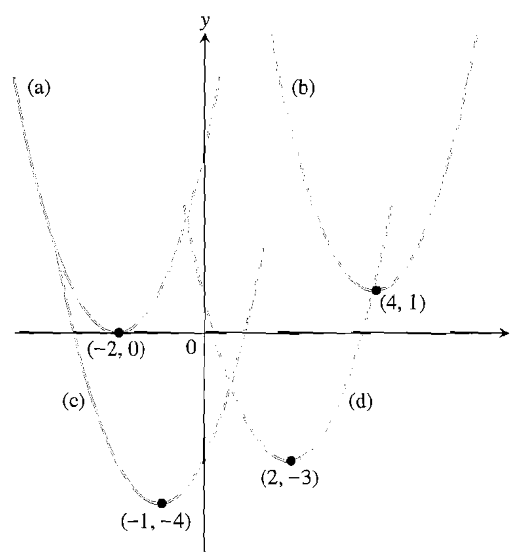
2. The graph of $y = x^2$ is shifted to two new positions. Write equations for the new graphs.



3. Match the equations listed to the graphs shown.
   a) $y + 4 = (x - 1)^2$ b) $y - 2 = (x - 2)^2$
   c) $y - 2 = (x + 2)^2$ d) $y + 2 = (x + 3)^2$



4. The graph of $y = x^2$ is shifted to four new positions. Write an equation for each new graph.



In Exercises 5–14, describe how each graph can be obtained from the graph of $f(x) = |x|$, $g(x) = 1/x$, $h(x) = \sqrt{x}$, or $k(x) = x^3$.

5. $y = |x + 4| - 3$
6. $y = |x - 3| + 2$
7. $y = 3\sqrt{-x}$
8. $y = -0.2\sqrt{-x}$
9. $y = \dfrac{2}{x} - 3$
10. $y = -\dfrac{0.5}{x} + 1$
11. $y = -0.5(x - 3)^3 + 1$
12. $y = 3\sqrt{2 - x} - 5$

13. $y = \dfrac{1}{x-2} + 3$

14. $y = \dfrac{2}{x-3} - 5$

In Exercises 15–24, sketch a complete graph of each function. Support your answer with a grapher. Determine the domain and range of the function.

15. $y = 4\sqrt[3]{2-x} - 5$

16. $y = 0.5|x+3| - 4$

17. $y = 5\sqrt[3]{-x} - 1$

18. $y = -2\sqrt[4]{1-x} + 3$

19. $y = -\dfrac{1}{(x+3)^2} + 2$

20. $y = \dfrac{1}{(x-2)^2}$

21. $y = -2(x-1)^{2/3} + 1$

22. $y = (x+2)^{3/2} + 2$

23. $y = 2[1-x]$

24. $y = [x-2] + 0.5$

Exercises 25–36 specify the order in which transformations are to be applied to the graph of the given equation. Give an equation for the transformed graph in each case. Support your answer with a grapher.

25. $y = x^2$, vertical stretch by 3, shift up 4

26. $y = x^2$, shift up 4, vertical stretch by 3

27. $y = \dfrac{1}{x}$, shift down 2, vertical shrink by 0.2

28. $y = \dfrac{1}{x}$, vertical shrink by 0.2, shift down 2

29. $y = |x|$, shift left 2, vertical stretch by 3, shift up 5

30. $y = |x|$, shift right 3, vertical shrink by 0.3, shift down 1

31. $y = x^3$, reflect through x-axis, vertical shrink by 0.8, shift right 1, shift down 2

32. $y = x^3$, vertical stretch by 2, reflect through x-axis, shift left 5, shift down 6

33. $y = \sqrt{x}$, reflect through y-axis, vertical stretch by 5, shift left 6, shift up 5

34. $y = \sqrt[4]{x}$, vertical shrink by 0.7, reflect through y-axis, shift right 8, shift down 7

35. $y = \sqrt{3x}$, horizontal stretch by 2, shift up 1

36. $y = 4|x|$, horizontal shrink by 0.5, shift down 3

37. Use a grapher to compare a vertical stretch of $y = x^2$ by a factor of 4 with a horizontal shrink by a factor of 0.5. Algebraically confirm your observation. Generalize.

38. Use a grapher to compare a vertical stretch of $y = |x|$ by a factor of 2 with a horizontal shrink by a factor of 0.5. Algebraically confirm your observation. Generalize.

39. Are the graphs of the equations found in Exercises 25 and 26 the same? Explain any differences.

40. Are the graphs of the equations found in Exercises 27 and 28 the same? Explain any differences.

41. Let $f(x) = \sqrt[3]{x}$.
   a) Describe how the graph of $y = \sqrt[3]{-x}$ can be obtained from the graph of f.
   b) Describe how the graph of $y = -\sqrt[3]{x}$ can be obtained from the graph of f.
   c) Compare the graphs of the functions in parts (a) and (b). Explain any similarities or differences.

42. Let $f(x) = \sqrt{x}$.
   a) Describe how the graph of $y = \sqrt{-x}$ can be obtained from the graph of f.
   b) Describe how the graph of $y = -\sqrt{x}$ can be obtained from the graph of f.
   c) Compare the graphs of the functions in parts (a) and (b). Explain any similarities or differences.

43. The line $y = mx$ is shifted vertically to make it pass through the point $(0, b)$. What is the line's new slope-intercept equation?

44. The line $y = mx$ is shifted horizontally and vertically to make it pass through the point (x_0, y_0). What is the line's new point-slope equation?

In Exercises 45–48, the graph of g is obtained by applying, in order, the given transformations to the graph of $f(x) = |x|$. Find the points on the graph of g that correspond under the sequence of transformations to the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$ on the graph of f.

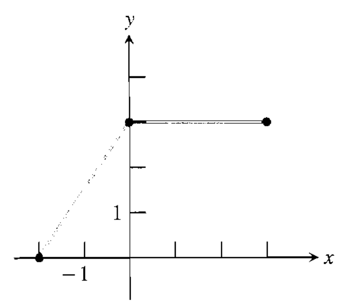
45. Shift right 3, shift up 2

46. Shift down 4, shift left 1

47. Vertical stretch by 2, reflect through x-axis

48. Vertical shrink by 0.3, shift up 4

A complete graph of f is pictured. Sketch a complete graph of each function in Exercises 49–56.



49. $y = f(-x)$

50. $y = -f(x)$

51. $y = f(x-2)$

52. $y = f(x+3)$

53. $y = 0.5f(x) - 3$

54. $y = -3f(x) + 2$

55. $y = -2f(x+1) + 3$

56. $y = 0.2f(x-1) - 1$

57. Sketch a complete graph of $y = \dfrac{2x+1}{x-2}$. (*Hint:* Use long division to write $y = 2 + \dfrac{5}{x-2}$).

58. Sketch a complete graph of $y = \dfrac{x+1}{x+3}$. (*Hint:* See Exercise 57).

In Exercises 59–62, determine the vertex and axis of symmetry, and sketch a complete graph of each parabola. Support your work with a grapher.

59. $y = -2x^2 + 12x - 11$ **60.** $y = 3x^2 + 12x + 7$

61. $y = 4x^2 + 20x + 19$ **62.** $y = -4x^2 + 12x - 3$

In Exercises 63–68, use completing the square to rewrite the equation in the form $x = a(y+h)^2 + k$. Then describe how the graph of the equation can be obtained from the graph of $x = y^2$ and sketch a complete graph. Support your work by graphing the equation (a) in parametric mode and (b) in function mode using two functions of y.

63. $x = y^2 - 6y + 11$ **64.** $x = y^2 + 4y + 1$

65. $x = 2y^2 + 4y + 1$ **66.** $x = -3y^2 + 12y - 7$

67. $x = -2y^2 + 12y - 13$ **68.** $x = 4y^2 + 16y + 9$

69. Let $f(x) = x^2$ and suppose g is obtained by applying the following transformations in some order to f: shift up 3, shift right 2, reflect through the y-axis. How many different graphs can be obtained for g?

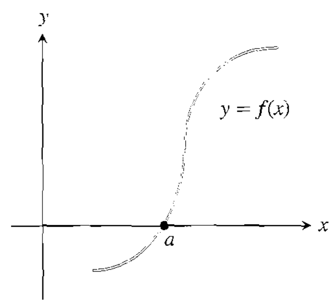
1.5 Solving Equations and Inequalities Graphically

ZOOM

Some graphing utilities have built-in procedures for solving equations (SOLVE, ROOT, and so on). We describe the ZOOM procedure here in order to discuss accuracy of solutions. Although the ZOOM feature itself uses various commands, such as ZOOM-IN, ZOOM-BOX, ZOOM-OUT, and so on, we will use the single command ZOOM-IN to indicate magnifying within our viewing window. Consult your *Resource Manual* or your *Owner's Guide* for more information about ZOOM and any explicit equation-solving features.

The standard algebraic techniques for solving equations and inequalities, such as the quadratic formula and factoring, are really not very useful for solving most equations or inequalities (ones like $x^3 - 2x^2 - 7 = 0$, $\sin x > x$, etc.). In this section, we show how to use a graphing utility to solve equations and inequalities graphically with prescribed accuracy.

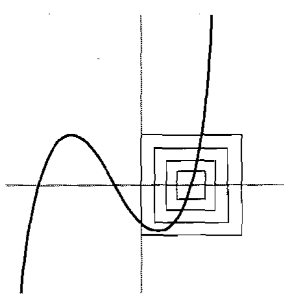
Unless we state otherwise, **solving an equation** will mean finding all its real number solutions. Every equation involving a single variable x can be put in the equivalent form $f(x) = 0$. To use the graph of $y = f(x)$ to help solve the equation $f(x) = 0$, we must find where the graph crosses the x-axis, that is, where $y = 0$ (Fig. 1.64).



1.64 The real number $x = a$ is a solution to the equation $f(x) = 0$ if and only if the point $(a, 0)$ is on the graph of $y = f(x)$.

Solving Equations Graphically Using ZOOM

A grapher procedure called ZOOM can be used to find solutions of equations to a high degree of accuracy regardless of the complexity of the equation. The idea is to "trap" the x-intercept of $y = f(x)$ in a decreasing sequence of viewing rectangles, each new one contained within the previous one, until the last viewing rectangle has enlarged a small enough portion of the graph so that the value of x can be read to the accuracy desired and within the limits of machine precision (Fig. 1.65).



1.65 ZOOM-IN or ZOOM-BOX creates a nested sequence of rectangles that fill our viewing windows with magnifications of the tiny region we are interested in exploring.