EXPLORATION 5

The Consumer Price Index

1. How long does it take the cost of living to increase 50%? To answer the question, assume a steady inflation rate of 4%, and find how long it takes the CPI to increase from p_0 to $1.5p_0$. This is equivalent to asking: When does $p_0e^{0.04t} = 1.5p_0$, or $e^{0.04t} = 1.5$? Find an answer graphically, then confirm algebraically.
2. GRAPH $y_1 = e^{rt}$ for $r = 0.03, 0.04, 0.06$, and 0.10 and $y_2 = 1.5$. Explain what you see in the viewing window in terms of the CPI model.

The Purchasing Power of the Dollar

The higher the Consumer Price Index, the less a dollar will buy. Economists call the number $100/p$ the **purchasing power of the dollar.**

In 1984, when the CPI was 100, the purchasing power of the dollar was $100/100 = 1.00$. Three years later, in 1987, it was $100/112.1 = 0.89$, and a year after that, it was $100/116.5 = 0.86$.

EXAMPLE 18

Assuming the 4% inflation rate of Example 17, what will the purchasing power of the dollar be in 1998?

Solution We divide the CPI for 1998 into 100:

$$\frac{100}{\text{1998 CPI}} = \frac{100}{173.8} = 0.58. \qquad \text{CPI value from Example 17}$$

This represents a change of -0.42, or a loss in purchasing power of 42% since 1984.

Exercises 7.2

Use the fact that the functions $y = \ln x$ and $y = e^x$ are inverses of each other to simplify the expressions in Exercises 1–6.

1. $e^{\ln 7}$ **2.** $e^{-\ln 7}$

3. $\ln e^2$ **4.** $e^{3\ln 2}$

5. $e^{2+\ln 3}$ **6.** $e^{-2\ln 3}$

In Exercises 7–12, solve for k.

7. $e^{2k} = 4$ **8.** $e^{5k} = \frac{1}{4}$

9. $100e^{10k} = 200$ **10.** $100e^k = 1$

11. $2^{k+1} = 3^k$ **12.** $\sqrt{4^{k-1}} = 3^k$

In Exercises 13–18, solve for t.

13. $e^t = 1$ **14.** $e^{kt} = \frac{1}{2}$

15. $e^{0.3t} = 27$ **16.** $e^{-0.01t} = 1000$

17. $2^{e^t} = 2 - t$ **18.** $e^{-2^t} = t - 2$

In Exercises 19–24, solve for y.

19. $\ln y = 2t + 4$ **20.** $\ln y = -t + 5$

21. $\ln(y - 40) = 5t$ **22.** $\ln(1 - 2y) = t$

23. $5 + \ln y = 2^{x^2+1}$ **24.** $\ln(2^y - 1) = x^2 - 3$

Find dy/dx in Exercises 25–34. Support graphically.

25. $y = 2e^x$ **26.** $y = e^{2x}$

27. $y = e^{-x}$ **28.** $y = e^{-5x}$

29. $y = e^{2x/3}$ **30.** $y = e^{-x/4}$

31. $y = xe^2 - e^x$ **32.** $y = x^2e^x - xe^x$

33. $y = e^{\sqrt{x}}$ **34.** $y = e^{(x^2)}$

Evaluate the integrals in Exercises 35–48. Support using a NINT computation.

35. $\int_1^{e^2} \frac{1}{x}\,dx$ 36. $\int_1^{e} \frac{2}{x}\,dx$

37. $\int_{\ln 2}^{\ln 3} e^x\,dx$ 38. $\int_{-1}^{1} e^{(x+1)}\,dx$

39. $\int_{\ln 3}^{\ln 5} e^{2x}\,dx$ 40. $\int_0^{\ln 2} e^{-x}\,dx$

41. $\int_0^1 (1+e^x)e^x\,dx$ 42. $\int_{-1}^{1} \frac{e^x}{1+e^x}\,dx$

43. $\int_2^4 \frac{dx}{x+2}$ 44. $\int_{-1}^{0} \frac{8\,dx}{2x+3}$

45. $\int_{-1}^{1} 2xe^{-x^2}\,dx$ 46. $\int_0^1 \frac{x\,dx}{4x^2+1}$

47. $\int_1^4 \frac{e^{\sqrt{x}}\,dx}{2\sqrt{x}}$ 48. $\int_e^{e^2} \frac{dx}{x\ln x}$

Evaluate the integrals in Exercises 49–54.

49. $\int 2e^x \cos(e^x)\,dx$ 50. $\int 3e^x \sin\left(e^x\right)\,dx$

51. $\int \frac{e^x\,dx}{1+e^x}$ 52. $\int \frac{dx}{1-e^{-x}}$

53. $\int \frac{\tan\left(\sqrt{x}\right)\,dx}{\sqrt{x}}$ 54. $\int \frac{\cot\left(\sqrt{2x}\right)\,dx}{\sqrt{x}}$

In Exercises 55–58:

a) Find the inverse f^{-1} of the function f, expressed as a function of x.

b) Graph f and f^{-1} together.

c) Verify Eq. (3) by evaluating df/dx at $x=a$ and df^{-1}/dx at $x=f(a)$.

55. $f(x)=2x+3, \quad a=-1$

56. $f(x)=5-4x, \quad a=1/2$

57. $f(x)=\dfrac{1-2x}{x+2}, \quad a=-3$

58. $f(x)=\dfrac{x-5}{x-3}, \quad a=2$

One of the virtues of Eq. (3) is that it enables us to find values of df^{-1}/dx even when we do not have an explicit formula for the derivative. As a case in point, let $f(x)=x^2-4x-3$, $x>2$, and find the value of df^{-1}/dx at the point specified in Exercises 59 and 60.

59. $x=-3=f(4)$ 60. $x=2=f(5)$

61. Let $f(x)=\dfrac{2x+3}{x+3}$. Verify the claims in Example 6 that $f'(-5)=3/4$ and $(f^{-1})'(3.5)=4/3$.

Find the limits in Exercises 62–65.

62. $\lim_{x\to\infty} e^{-x}$ 63. $\lim_{x\to-\infty} e^{-x}$

64. $\lim_{x\to-\infty} \ln(2+e^x)$ 65. $\lim_{x\to\infty} \int_x^{2x} \frac{1}{t}\,dt$

66. Solve for x: $\int_0^x \frac{1}{\sqrt{2\pi}} e^{(-t^2/2)}\,dt = 0.3$

67. Solve for x: $\int_1^x 2^t \ln t\,dt = 0.1$

68. Find the linearization of $f(x)=e^x$ at $x=0$.

69. Find the linearization of $f(x)=x+e^{4x}$ at $x=0$.

70. Find the maximum value of $f(x)=x^2\ln(1/x)$.

71. Find the maximum and minimum values of the periodic function $f(x)=e^{\sin x}$.

Solve the initial value problems in Exercises 72 and 73.

72. Differential equation: $\dfrac{dy}{dx}=(\cos x)e^{\sin x}$

Initial condition: $y=0$ when $x=0$

73. Differential equation: $\dfrac{dy}{dx}=1+\dfrac{1}{x}$

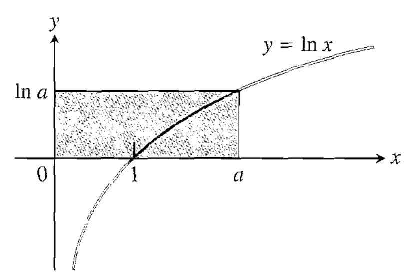
Initial condition: $y=3$ when $x=1$

74. A body moves along a coordinate line with acceleration $d^2s/dt^2=4/(4-t)^2$. When $t=0$, the body's velocity is 2 m/sec. Find the total distance traveled by the body from time $t=1$ sec to time $t=2$ sec.

75. Show that, for any number $a>1$,

$$\int_1^a \ln x\,dx + \int_0^{\ln a} e^y\,dy = a\ln a.$$

(*Hint:* Study this diagram.)



76. *Rules of Exponents.* Let $e^{x_1}=e^{x_1-x_2+x_2}$ and use the first rule of exponents to show that $e^{x_1}/e^{x_2}=e^{x_1-x_2}$.

77. *Rules of Exponents.* Use the second rule of exponents to prove that $e^{-x}=1/e^x$.

78. *Cholera bacteria.* Suppose that the bacteria in a colony can grow unchecked, by the Law of Exponential Change. The colony starts with 1 bacterium and doubles every half hour. How many bacteria will the colony contain at the end of 24 hr? (Under favorable laboratory conditions, the number