Solution The following inequalities are equivalent provided that $x \neq 2$:

$$\left|\frac{3}{x-2}\right| < 1$$

$$\frac{3}{|x-2|} < 1 \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$\frac{|x-2|}{3} > 1 \qquad \text{Taking reciprocals reverses the inequality.}$$

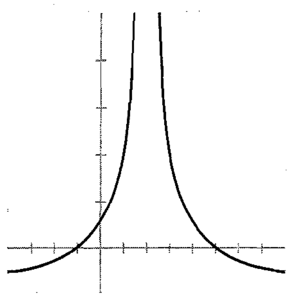
$$|x-2| > 3 \qquad \text{Multiply by 3.}$$

$$x-2 < -3 \text{ or } x-2 > 3 \qquad \text{Equation (4) with } a = x-2 \text{ and } D = 3.$$

$$x < -1 \text{ or } x > 5 \qquad \text{Adding 2.}$$

The original inequality holds if and only if $x < -1$ or $x > 5$.

The graph of $y = |3/(x-2)| - 1$ in Fig. 1.76 supports the solution because it appears to be below the x-axis for x in the intervals $(-\infty, -1)$ and $(5, \infty)$. ∎



[−4, 8] by [−1, 5]

1.76 A complete graph of $y = |3/(x-2)| - 1$.

EXAMPLE 12

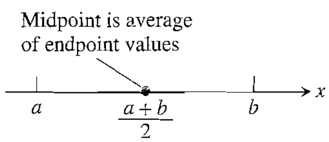
Describe the interval $-3 < x < 5$ with an absolute value inequality of the form $|x - x_0| < D$.

Solution We average the endpoint values to find the interval's midpoint (Fig. 1.77):

$$\text{midpoint } x_0 = \frac{-3+5}{2} = \frac{2}{2} = 1.$$

The midpoint lies 4 units from each endpoint. The interval therefore consists of the points that lie within 4 units of the midpoint, or the points x with

$$|x - 1| < 4.$$ ∎



1.77 The midpoint value in the interval (a, b) is found by averaging the endpoint values.

Exercises 1.5

Solve the equations in Exercises 1–4 algebraically. Support your answers graphically.

1. $6x^2 + 5x - 6 = 0$
2. $x^2 - 4x + 1 = 0$
3. $4x^3 - 16x^2 + 15x + 2 = 0$
4. $x^3 - x^2 + x = 0$

Solve the equations in Exercises 5–8 graphically. Confirm your answers algebraically.

5. $2x^2 + 7x - 4 = 0$
6. $9x^2 - 6x = 4$
7. $18x^3 - 3x^2 - 14x = 4$
8. $x^3 - 2x^2 + 2x = 0$

In Exercises 9–10, find a sequence of four viewing windows containing each solution. Choose each sequence to permit the solutions to be read with errors of at most 0.1, 0.01, 0.001, and 0.0001.

9. $x^3 - 2x^2 - 5x + 5 = 0$
10. $x^3 - 4x + 1 = 0$

Solve the equations in Exercises 11–16.

11. $9x^2 - 6x + 5 = 0$
12. $x^2 + x - 12 = 0$
13. $2x^3 - 8x^2 + 3x + 9 = 0$
14. $10 + x + 2x^2 = x^3$
15. $x^3 - 21x^2 + 111x = 71$
16. $x^3 + 19x^2 + 90x + 52 = 0$

17. If $2 < x < 6$, which of the following statements about x are true and which are false?

a) $0 < x < 4$ **b)** $0 < x - 2 < 4$

c) $1 < \frac{x}{2} < 3$ **d)** $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$

e) $1 < \frac{6}{x} < 3$ **f)** $|x - 4| < 2$

g) $-6 < -x < 2$ **h)** $-6 < -x < -2$

18. If $-1 < y - 5 < 1$, which of the following statements about y are true and which are false?

a) $4 < y < 6$ **b)** $|y - 5| < 1$

c) $y > 4$ **d)** $y < 6$

e) $0 < y - 4 < 2$ **f)** $2 < \frac{y}{2} < 3$

g) $\frac{1}{6} < \frac{1}{y} < \frac{1}{4}$ **h)** $-6 < y < -4$

Solve the equations in Exercises 19–24.

19. $|x| = 2$ **20.** $|x - 3| = 7$

21. $|2x + 5| = 4$ **22.** $|1 - x| = 1$

23. $|8 - 3x| = 9$ **24.** $\left|\frac{x}{2} - 1\right| = 1$

The inequalities in Exercises 25–30 define intervals. Describe each interval with inequalities that do not involve absolute values.

25. $|y - 1| \le 2$ **26.** $|y + 2| < 1$

27. $|3y - 7| < 2$ **28.** $\left|\frac{y}{3}\right| \le 10$

29. $|1 - y| < \frac{1}{10}$ **30.** $\left|\frac{7 - 3y}{2}\right| < 1$

Describe the intervals in Exercises 31–34 with absolute value inequalities of the form $|x - x_0| < D$. It may help to draw a picture of the interval first.

31. $3 < x < 9$ **32.** $-3 < x < 9$

33. $-5 < x < 3$ **34.** $-7 < x < -1$

Solve the inequalities in Exercises 35–38 algebraically. Support your answers graphically.

35. $|x - 5| < 2$ **36.** $\left|\frac{3x}{2}\right| < 5$

37. $\left|\frac{4}{x - 1}\right| \le 2$ **38.** $|3x + 2| > 3$

Solve the inequalities in Exercises 39–44 graphically. Confirm your answers algebraically.

39. $|x + 3| \le 5$ **40.** $\left|\frac{2x}{5}\right| \le 1$

41. $\left|\frac{2}{x + 3}\right| < 1$ **42.** $|3x + 2| \ge 1$

43. $|2 - 3x| < 4$ **44.** $\left|5 - \frac{x}{2}\right| \le 1$

Solve the inequalities in Exercises 45–50.

45. $x^2 + 3x - 10 \le 0$ **46.** $4x^2 - 8x + 5 > 0$

47. $x^3 - 6x^2 + 5x + 6 \le 0$

48. $x^3 - 2x^2 - 5x + 20 > 0$

49. $x^3 - 4x^2 + 3.99x > 0$

50. $-x^3 + 0.2x^2 + 18.14x > -28.8$

Use ZOOM-IN to find the coordinates of the vertex of each parabola in Exercises 51–52, with an error of at most 0.01. Confirm your answers algebraically.

51. $y = 30x - x^2$ **52.** $y = -x^2 + 22x - 21$

53. Consider the function $f(x) = x^3 - 2x^2 - 1$ of Example 1.

a) Draw the graph of f in the following viewing windows: $[-5, 5]$ by $[-5, 5]$, $[2, 3]$ by $[-5, 5]$, $[2.2, 2.3]$ by $[-5, 5]$, and $[2.2, 2.3]$ by $[-0.1, 0.1]$.

b) Explain why it is necessary to change yMin and yMax when we ZOOM-IN on an x-intercept.

54. Solve the equation $x^3 - 2x = 2\cos x$.

55. Explain how the Rational Zeros Theorem can be applied to a polynomial equation with noninteger rational coefficients such as $2x^3 + \frac{1}{2}x^2 - \frac{2}{3}x + 1 = 0$.

56. Explain why $ax^3 + bx^2 + cx + d = 0$ (a, b, c, d real, $a \neq 0$) will always have at least one real zero. Explain how to find all real zeros of any cubic polynomial equation.

57. One hundred feet of fencing is used to enclose a rectangular garden. Let x be the length of one side of the garden.

a) Find an algebraic representation $A(x)$ for the area of the garden.

b) Draw a complete graph of $y = A(x)$.

c) What are the domain and range of $y = A(x)$?

d) What values of x make sense in the problem situation? Draw a graph of the problem situation.

e) Use a graph to determine the dimensions of the garden if the area is 500 ft^2. Confirm your answer algebraically.

f) Determine the possible values of x if the area of the garden is to be less than 500 ft^2.

58. One hundred feet of fencing is used to enclose three sides of a rectangular pasture. The side of a barn closes off the fourth side. Let x be the length of one side of the fence perpendicular to the barn.

a) Find an algebraic representation $A(x)$ for the area of the pasture.

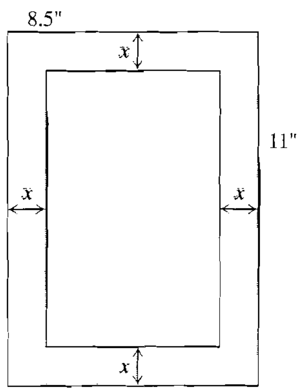
b) Draw a complete graph of $y = A(x)$.

c) What are the domain and range of $y = A(x)$?

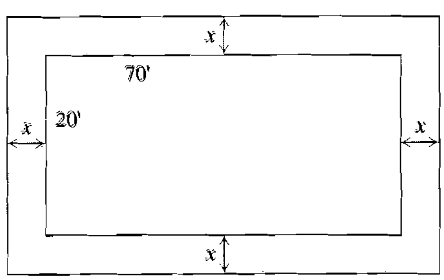
d) What values of x make sense in the problem situation? Draw a graph of the problem situation.

e) Use a graph to determine the dimensions of the pasture if the area is 500 ft^2. Confirm your answer algebraically.
f) Determine the possible values of x if the area of the pasture is to be less than 500 ft^2.

59. An 8.5- by 11-in. piece of paper contains a picture with uniform border. The distance from the edge of the paper to the picture is x inches on all sides.
a) Write the area A of the picture as a function of x.
b) Draw a complete graph of $y = A(x)$.
c) What are the domain and range of $y = A(x)$?
d) What values of x make sense in the problem situation? Which portion of the graph in part (b) is a graph of the problem situation?
e) Determine the width of the border if the area of the picture is 60 in^2.



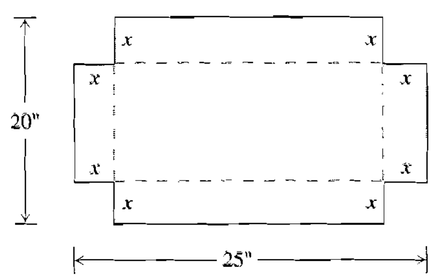
60. A 20- by 70-ft swimming pool is surrounded by a walk of uniform width. The distance from the outer edge of the walk to the pool is x feet on all sides.
a) Write the area A of the sidewalk as a function of x.
b) Draw a complete graph of $y = A(x)$.
c) What are the domain and range of $y = A(x)$?
d) What values of x make sense in the problem situation? Which portion of the graph in part (b) is a graph of the problem situation?
e) Determine the width of the sidewalk if its area is 500 ft^2.



61. A money box contains 50 coins in dimes and quarters. Let x be the number of dimes in the box.
a) Find an algebraic representation $V(x)$ for the value of the coins in the box.
b) Draw a complete graph of $y = V(x)$.
c) What are the domain and range of $y = V(x)$?
d) What values of x make sense in the problem situation? Which portion of the graph in part (b) is a graph of the problem situation?
e) Determine the number of each coin in the box if the value of the coins is \$9.20.
f) Repeat part (e) if the value of the coins is \$6.25.

62. Sherrie invests \$10,000, part at 6.5% simple interest and the remainder at 8% simple interest. Let x be the amount she invests at 6.5% interest.
a) Find an algebraic representation $I(x)$ for the total interest received in one year.
b) Draw a complete graph of $y = I(x)$.
c) What are the domain and range of $y = I(x)$?
d) What values of x make sense in the problem situation? Which portion of the graph in part (b) is a graph of the problem situation?
e) Determine the amount invested at each rate if Sherrie receives \$766.25 interest in one year.

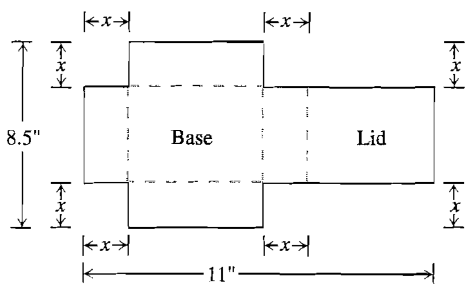
63. *The open box problem.* Equal squares of side length x are removed from each corner of a 20- by 25-in. piece of cardboard. The sides are turned up to form a box with no top.
a) Write the volume V of the box as a function of x.
b) Draw a complete graph of $y = V(x)$.
c) What are the domain and range of $y = V(x)$?
d) What values of x make sense in the problem situation? Draw a graph of the problem situation.
e) Use ZOOM-IN to determine x so that the resulting box has maximum possible volume. What is the maximum possible volume?



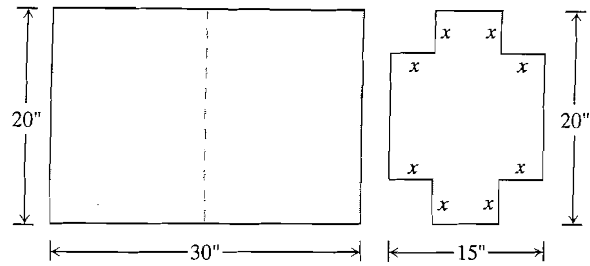
64. Repeat Exercise 63 for a 25- by 30-in. piece of cardboard.

65. *The box with lid problem.* Two congruent squares are removed from one end of a rectangular 8.5- by 11-in. piece of cardboard. Two congruent rectangles are removed from the other end as shown.

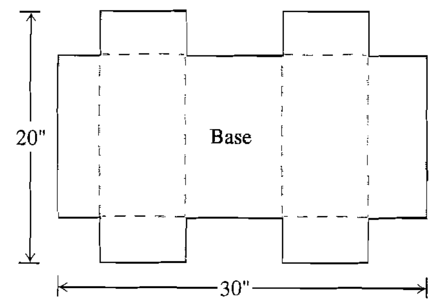
a) Determine the length of the two rectangles to be removed so that when the cardboard is folded along the dashed lines, the box formed has a lid that exactly covers the top.
b) Find an algebraic representation $V(x)$ for the volume of the box with lid.
c) Draw a complete graph of $y = V(x)$. What are the domain and range of the function?
d) What values of x make sense in the problem situation? Draw a graph of the problem situation.
e) Determine the side length x of the squares to remove to construct a box with volume 25 in.^{3}
f) Determine x so that the resulting box has maximum volume. What is the maximum volume?



66. *The suitcase box with lid problem.* A rectangular 20- by 30-in. sheet of cardboard is folded in half to form a 20- by 15-in. rectangle from which four congruent corner squares of side length x are removed.



The sheet is then unfolded.



a) Show that folding along the dashed lines forms a box with lid with volume $V(x) = 2x(20 - 2x)(15 - 2x)$.
b) Draw a complete graph of $y = V(x)$. What are the domain and range?
c) What values of x make sense in the problem situation? Draw a graph of the problem situation.
d) Determine x so that the resulting box has volume 300 in^3.
e) Determine x so that the resulting box has maximum volume. What is the maximum volume?

1.6 Relations, Functions, and Their Inverses

We recall from Section 1.3 that a relation is simply a set of ordered pairs. A function is a relation with a special condition, namely that no two of its ordered pairs can have the same first element. A nice example of a relation that is *not* a function is one whose graph is a circle, which clearly has a pair of points (many pairs, in fact) with the same first coordinate.

Equations for Circles in the Plane

DEFINITION

A **circle** is the set of points in a plane whose distance from a fixed point in the plane is a constant. The fixed point is the **center** of the circle. The constant distance is the **radius** of the circle.