EXPLORATION BIT

In Example 12, we compute $(f \circ g)(2) = f(g(2))$. Now you compute $(g \circ f)(2)$ and compare with $(f \circ g)(2)$.

In the notation for composite functions, the parentheses tell which function comes first:

The notation $f(g(x))$ says "first g, then f." To calculate $f(g(2))$, calculate $g(2)$ and then apply f.

The notation $g(f(x))$ says "first f, then g." To calculate $g(f(2))$, calculate $f(2)$ and then apply g.

EXPLORATION 7

Composing Functions

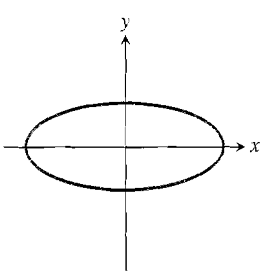
Some graphers allow a function such as y_1 to be used as a variable (argument) in another function. In other words, we can compose functions.

1. Enter the functions $g(x) = x^2$ and $f(x) = x - 7$ of Example 12 as $y_1 = x^2$ and $y_2 = x - 7$. Enter $y_3 = y_1 - 7$ and $y_4 = {y_2}^2$. Which of y_3 and y_4 corresponds to $f \circ g$? To $g \circ f$?
2. GRAPH y_3 and y_4. Make a conjecture about $f \circ g$ and $g \circ f$. Confirm your conjecture algebraically by finding formulas for $f(g(x))$ and $g(f(x))$.
3. Explore function composition on your own. Investigate such questions as the following. Feel free to create your own questions.
   a) Are there two functions f and g so that $f \circ g = g \circ f$?
   b) For a given function f, is there a function g so that $f \circ g = g \circ f$?
   c) Are there two functions f and g with nonstraight graphs for which the graph of $f \circ g$ is straight?
   d) What is a reasonable definition of composition $f \circ g \circ h$ for three functions? For four functions?

Exercises 1.3

In Exercises 1–4, identify which of the relations graphed are functions. Explain your answer.

1.



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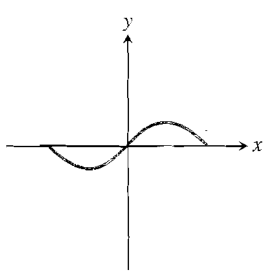

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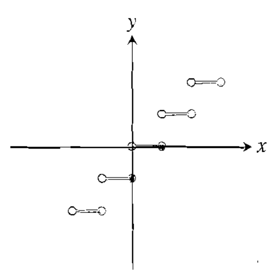

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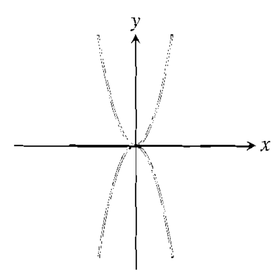


For each given point $P(x, y)$ in Exercises 5–12, use symmetry tests to find the point Q that is (a) symmetric to P across the x-axis, (b) symmetric to P across the y-axis, and (c) symmetric to P through the origin.

5. $(3, 1)$ **6.** $(-2, 2)$

7. $(-2, 1)$ **8.** $(-1, -1)$

9. $(1, -\sqrt{2})$ **10.** $(-\sqrt{3}, -\sqrt{3})$

11. $(0, \pi)$ **12.** $(2, 0)$

In Exercises 13–20, find the domain and range of each function. Support your answer with a graphing utility.

13. $y = 2 + \sqrt{x - 1}$ **14.** $y = -3 + \sqrt{x + 4}$

15. $y = -\sqrt{-x}$ **16.** $y = \sqrt{-x}$

17. $y = 2\sqrt{3 - x}$ **18.** $y = -3\sqrt{2 - x}$

19. $y = \dfrac{1}{x - 2}$ **20.** $y = \dfrac{1}{x + 2}$

Use a graph to find the domain and range of each function in Exercises 21–30. What symmetries described in this section, if any, does each graph have?

21. $y = x^2 - 9$ **22.** $y = 4 - x^2$

23. $y = \sqrt[3]{x - 3}$ **24.** $y = -2\sqrt[3]{x + 2}$

25. $y = 1 + \sqrt[3]{2 - x}$ **26.** $y = -2 + 5\sqrt[3]{4 - x}$

27. $y = -\dfrac{1}{x}$ **28.** $y = -\dfrac{1}{x^2}$

29. $y = 1 + \dfrac{1}{x}$ **30.** $y = 1 + \dfrac{1}{x^2}$

31. Consider the function $y = 1/\sqrt{x}$.
a) Can x be negative?
b) Can $x = 0$?
c) What is the domain of the function?

32. Consider the function $y = \sqrt{(1/x) - 1}$.
a) Can x be negative?
b) Can $x = 0$?
c) Can x be greater than 1?
d) What is the domain of the function?

Determine whether each function in Exercises 33-42 is even, odd, or neither. Try to answer without writing anything (except the answer).

33. $y = x^3$ **34.** $y = x^4$

35. $y = x + 2$ **36.** $y = x + x^2$

37. $y = x^2 - 3$ **38.** $y = x + x^3$

39. $y = \dfrac{1}{x^2 - 1}$ **40.** $y = \dfrac{1}{x - 1}$

41. $y = \dfrac{x}{x^2 - 1}$ **42.** $y = \dfrac{x^2}{x^2 - 1}$

Test each equation in Exercises 43–50 to find what symmetries its graph has. Then draw a complete graph of the equation.

43. $y = -x^2$ **44.** $x = 4 - y^2$

45. $y = 1/x^2$ **46.** $y = 1/(x^2 + 1)$

47. $xy = 1$ **48.** $xy^2 = 1$

49. $x^2y^2 = 1$ **50.** $x^2 + 4y^2 = 1$

Draw a complete graph of each function in Exercises 51–62.

51. $y = |x + 3|$ **52.** $y = |2 - x|$

53. $y = \dfrac{|x|}{x}$ **54.** $y = \dfrac{|x - 1|}{x - 1}$

55. $y = \dfrac{x - |x|}{2}$ **56.** $y = \dfrac{x + |x|}{2}$

57. a) $f(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & 1 < x \end{cases}$
b) Compute $f(0), f(1), f(2.5)$.

58. a) $f(x) = \begin{cases} 1/x, & x < 0 \\ x, & 0 \le x \end{cases}$
b) Compute $f(-1), f(0), f(\pi)$.

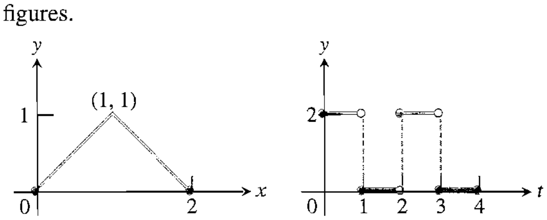
59. a) $f(x) = \begin{cases} 1, & x < 5 \\ 0, & 5 \le x \end{cases}$
b) Compute $f(0), f(5), f(6)$.

60. a) $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$
b) Compute $f(-1), f(0), f(5)$.

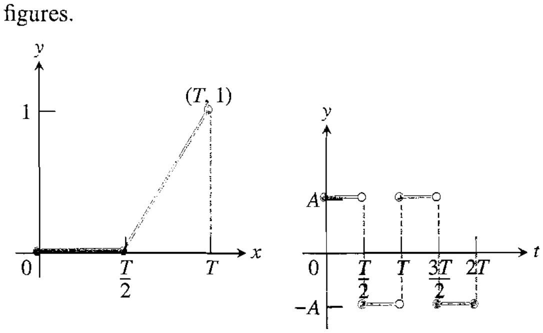
61. a) $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \dfrac{3}{2}x + \dfrac{3}{2}, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$
b) Compute $f(0.5), f(1), f(3), f(4)$.

62. a) $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$
b) Compute $f(-1)$, $f(0)$, $f(1)$, $f(2.5)$.

63. Find formulas for the functions graphed in the following figures.



64. Find formulas for the functions graphed in the following figures.



65. For what values of x does (a) $[x] = 0$? (b) $\lceil x \rceil = 0$?

66. Does $[x]$ ever equal $\lceil x \rceil$? Explain.

67. Graph each function over the given interval.
 a) $y = x - [x]$, $-3 \le x \le 3$
 b) $y = [x] - \lceil x \rceil$, $-3 \le x \le 3$

68. *Integer parts of decimals.* When x is positive or zero, $[x]$ is the integer part of the decimal representation of x. What is the corresponding description of $\lceil x \rceil$ when x is negative or zero?

Draw a complete graph of each function in Exercises 69–72. Confirm algebraically by expressing the function without absolute value symbols.

69. $f(x) = |x + 1| + 2|x - 3|$ *Hint:* Use three intervals.

70. $f(x) = |x + 2| + |x - 1|$

71. $f(x) = |x| + |x - 1| + |x - 3|$ *Hint:* Use four intervals.

72. $f(x) = |x + 2| + |x| + |x + 1|$

In Exercises 73 and 74 find the domains of f and g; then find the corresponding domains and complete graphs of

$$f + g,\ f - g,\ f \cdot g,\ f/g, \text{ and } g/f.$$

73. $f(x) = x \quad g(x) = \sqrt{x - 1}$

74. $f(x) = \sqrt{x + 1}, \quad g(x) = \sqrt{x - 1}$

75. If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following:
 a) $f(g(0))$ b) $g(f(0))$
 c) $f(g(x))$ d) $g(f(x))$
 e) $f(f(-5))$ f) $g(g(2))$
 g) $f(f(x))$ h) $g(g(x))$

76. If $f(x) = x + 1$ and $g(x) = x - 1$, find the following:
 a) $f(g(0))$ b) $g(f(0))$
 c) $f(g(1))$ d) $g(f(1))$
 e) $f(g(x))$ f) $g(f(x))$

77. Copy and complete the following table.

	$g(x)$	$f(x)$	$f \circ g(x)$
a)	$x - 7$	$\sqrt{x}$	?
b)	$x + 2$	$3x$	?
c)	?	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
d)	$\dfrac{x}{x - 1}$	$\dfrac{x}{x - 1}$	?
e)	?	$1 + \dfrac{1}{x}$	x
f)	$\dfrac{1}{x}$	?	x

78. a) If $f(x) = 1/x$, find $f(x + 2) - f(2)$.
 b) If $F(t) = 4t - 3$, find $F(t + 1) - F(1)$.

79. The cost of producing x items of a certain product is $C(x) = 0.001x^3 - 0.05x^2 + 2.6x + 50$.
 a) What is the cost of producing ten items?
 b) What is the meaning of the difference $C(30) - C(20)$?

80. Compare the domains and ranges of the functions $y = \sqrt{x^2}$ and $y = (\sqrt{x})^2$.

81. Compare the graphs of $y = \sqrt{x^2}$ and $y = |x|$. Explain what you observe.

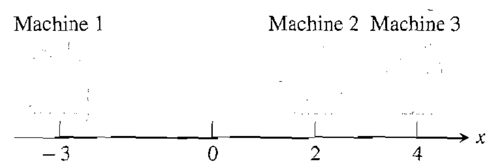
82. Find $f(x)$ if $g(x) = \sqrt{x}$ and $(g \circ f)(x) = |x|$.

83. Find $g(x)$ if $f(x) = x^2 + 2x + 1$ and $(g \circ f)(x) = |x + 1|$.

84. Find functions $f(x)$ and $g(x)$ whose composites satisfy the two equations

$$(g \circ f)(x) = |\sin x| \text{ and } (f \circ g)(x) = (\sin \sqrt{x})^2.$$

85. *The best location for a factory assembly table.* (adapted from *Fantastiks of Mathematiks*, Cliff Sloyer, Janson Publications, Inc., Providence, R.I., 1986.) Because of a design change, the parts produced by three machines along a factory aisle (shown here as the x-axis) are to go to a nearby table for assembly before they undergo further processing. Each assembly takes one part from each machine, and there is a fixed cost per foot for moving each part. As the plant's production engineer, you have been asked to find a location for the assembly table that will keep the total cost of moving the parts at a minimum.



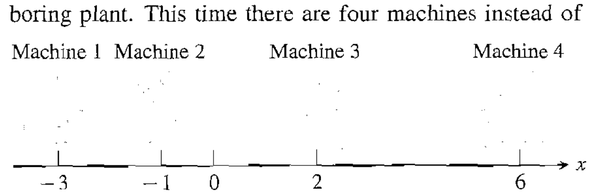
To solve the problem, you let x represent the table's location and look for the value of x that minimizes the sum

$$d(x) = |x + 3| + |x - 2| + |x - 4|$$

of the distances from the table to the three machines. Since the cost of moving the parts to the assembly table is proportional to the total distance the parts travel, any value of x that minimizes d will minimize the cost.

Complete the job now by graphing $d(x)$ to find its smallest value. Then say where you would put the table.

86. *Best location (continuation of Exercise 85).* You solved the table location problem in Exercise 85 so well that your manager has asked you to solve a similar problem at a neighboring plant. This time there are four machines instead of three and the cost is proportional to



$$d(x) = |x + 3| + |x + 1| + |x - 2| + |x - 6|.$$

Where should the assembly table go now?