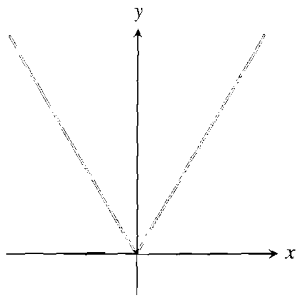
Exercises 1.6

Which of the functions graphed in Exercises 1–4 are one-to-one?

1.



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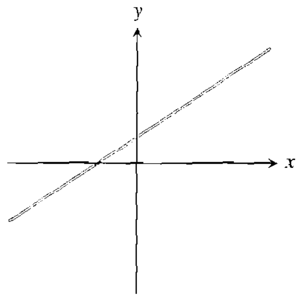

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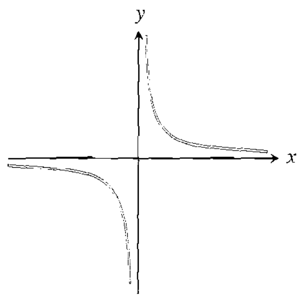

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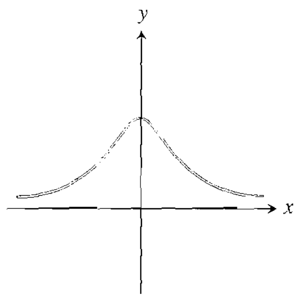
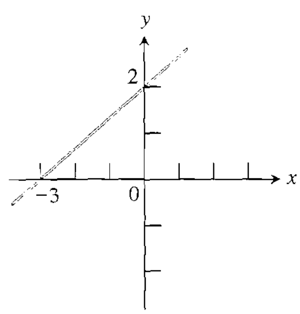


For each relation graphed in Exercises 5–8, sketch the graph of the inverse relation.

5.



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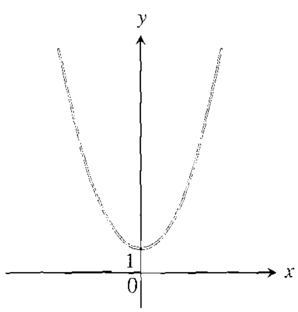

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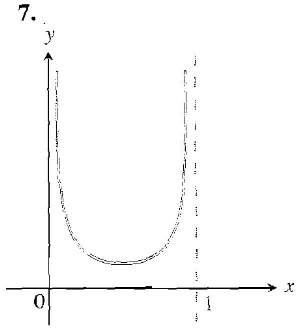

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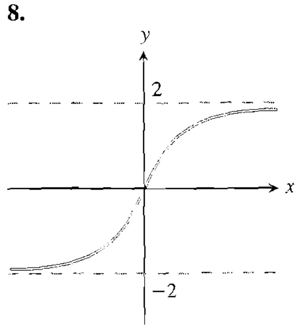


In Exercises 9–12, find an equation for the circle with the given center $C(h, k)$ and radius a. Then sketch a complete graph of the circle.

9. $C(0, 2), a = 2$

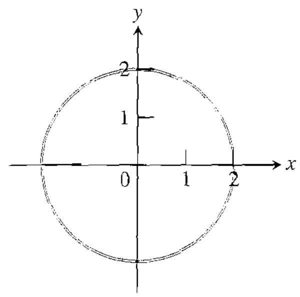
10. $C(-2, 0), a = 3$

11. $C(3, -4), a = 5$

12. $C(1, 1), a = \sqrt{2}$

Write equations for the circles in Exercises 13–16.

13.



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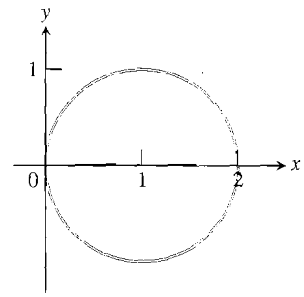

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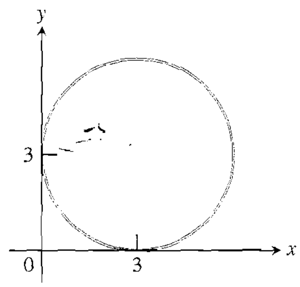

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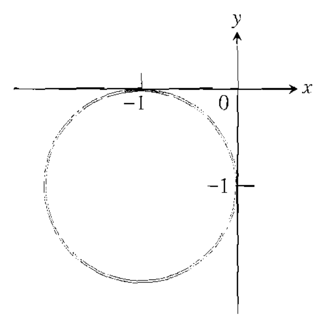

Find the center and radius of the circles in Exercises 17–20. Determine each domain and range. Sketch a complete graph.

17. $x^2 + y^2 - 6x + 8y = -16$

18. $x^2 + y^2 + 2x - 4y = 11$

19. $x^2 + y^2 + 4x + 6y + 8 = 0$

20. $x^2 + y^2 - 2x - 8y + 10 = 0$

Use a grapher to obtain complete graphs of the equations in Exercises 21 and 22.

21. $16x^2 - 9y^2 = 144$

22. $4x^2 + 9y^2 = 36$

Describe the regions defined by the inequalities and pairs of inequalities in Exercises 23 and 24.

23. **a)** $x^2 + y^2 > 1$
b) $x^2 + y^2 < 4$
c) the inequalities in parts (a) and (b) together

24. **a)** $x^2 + y^2 \geq 1$
b) $x^2 + y^2 \leq 4$
c) the inequalities in parts (a) and (b) together

25. Write an inequality that describes the points that lie inside the circle with center $C(-2, -1)$ and radius $a = \sqrt{6}$.

26. Write an inequality that describes the points that lie outside the circle with center $C(-4, 2)$ and radius $a = 4$.

Which of each function in Exercises 27–32 has an inverse that is also a function? Explain and demonstrate how you can support your answers with a grapher in parametric mode.

27. $y = \dfrac{3}{x-2} - 1$ **28.** $y = x^2 + 5x$

29. $y = x^3 - 4x + 6$ **30.** $y = x^3 + x$

31. $y = \ln x^2$ **32.** $y = 2^{3-x}$

In Exercises 33–44, find $f^{-1}(x)$ and show that $f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$. Draw a complete graph of f and f^{-1} in the same viewing window.

33. $f(x) = 2x + 3$ **34.** $f(x) = 5 - 4x$

35. $f(x) = x^3 - 1$ **36.** $f(x) = 2 - x^3$

37. $f(x) = x^2 + 1, x \geq 0$ **38.** $f(x) = x^2, x \leq 0$

39. $f(x) = -(x-2)^2, x \leq 2$

40. $f(x) = (x+1)^2, x \geq -1$

41. $f(x) = \dfrac{1}{x^2}, x > 0$ **42.** $f(x) = \dfrac{1}{x^3}, x \neq 0$

43. $f(x) = \dfrac{2x+1}{x+3}, x \neq -3$

44. $f(x) = \dfrac{x+3}{x-2}, x \neq 2$

Draw a complete graph of each function in Exercises 45–52.

45. $y = 2\log_3(x-4) - 1$ **46.** $y = -3\log_5(2-x) + 1$

47. $y = -3\log_{0.5}(x+2) + 2$

48. $y = 2\log_{0.2}(3-x) + 1$

49. $y = 5(e^{3x}) + 2$ **50.** $y = 3(e^{2-x}) - 1$

51. $y = -2(3^x) + 1$ **52.** $y = -5(2^{-x+1}) + 3$

Determine the domain and range and describe how the graphs of each equation in Exercises 53–58 can be obtained from the graph of an appropriate exponential function $y = a^x$, logarithmic function $y = \log_a x$, or circle $x^2 + y^2 = a^2$.

53. $y = -3\log(x+2) + 1$ **54.** $y = 2\ln(3-x) - 4$

55. $y = 2(3^{1-x}) + 1.5$ **56.** $y = -3(5^{x-2}) + 3$

57. $(x+3)^2 + (y-5)^2 = 9$

58. $(x-6)^2 + (y+1)^2 = 25$

In Exercises 59–62, graph f, f^{-1}, and the line $y = x$ in the same square viewing window.

59. $f(x) = 2^x$ **60.** $f(x) = 0.5^x$

61. $f(x) = \log_3 x$ **62.** $f(x) = \log_{0.3} x$

Solve the equations in Exercises 63–66.

63. $e^x + e^{-x} = 3$ **64.** $2^x + 2^{-x} = 5$

65. $\log_2 x + \log_2(4-x) = 0$

66. $\log x + \log(3-x) = 0$

67. Show that the graph of the inverse R^{-1} of a relation R can be obtained by the following two-step process: Rotate the graph of R 90° counterclockwise about the origin, and then reflect the resulting graph through the y-axis. (*Hint:* The path of a point under this process is $(a, b) \rightarrow (-b, a) \rightarrow (b, a)$.)

68. Prove Properties of Logarithms 6 and 7.

69. Use the Change of Base Formula to show that $\log_b a = \dfrac{1}{\log_a b}$

70. Suppose $a \neq 0, b \neq 1$, and $b > 0$. Determine the domain and range of each function.
a) $y = a(b^{c-x}) + d$
b) $y = a\log_b(x-c) + d$

1.7 A Review of Trigonometric Functions

In surveying, navigation, and astronomy, we measure angles in degrees, but in calculus, it is usually best to use radians. We will see why in Section 3.5. In the present section, we use radians and degrees together so that you can practice relating the two. We also review the trigonometry that you will need for calculus and its applications.

Radian Measure

The **radian measure** of the angle ACB at the center of the unit circle (Fig. 1.92) equals the length of the arc that the angle cuts from the unit circle.