Remark

The numbers e and π are **transcendental** as well as irrational numbers. A transcendental number is one that is *not* a root of a polynomial equation with integer coefficients. For example, $\sqrt{2}$ is irrational but is not transcendental since it is a root of the polynomial equation $x^2 - 2 = 0$. The number e was proved to be transcendental by the French mathematician Charles Hermite (1822–1901) in 1873, whereas π was proved to be transcendental nine years later by the German mathematician Ferdinand Lindemann. The latter proof showed conclusively that "squaring a circle" with a rule and a compass was impossible.

Exercises 8.2

Answers to odd-numbered problems begin on page A-32

In Problems 1–30 find the derivative of the given function.

1. $y = e^{-x}$
2. $y = e^{2x+3}$
3. $y = e^{\sqrt{x}}$
4. $y = e^{\cos 10x}$
5. $y = \dfrac{e^{-2x}}{x}$
6. $y = x^3 e^{4x}$
7. $y = \sqrt{1 + e^{-5x}}$
8. $y = \dfrac{1}{(e^{2x} - e^{-2x})^2}$
9. $y = \ln(x^4 + e^{x^2})$
10. $y = \ln\sqrt{e^x + e^{-x}}$
11. $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
12. $y = \dfrac{e^{7x}}{e^{-x}}$
13. $y = e^{\ln x}$
14. $y = \ln e^x$
15. $y = e^{3x}\ln(x^2 + 1)$
16. $y = \dfrac{\ln x}{e^x}$
17. $y = e^{\frac{x+2}{x-2}}$
18. $y = \ln\left|\dfrac{1 + e^{2x}}{1 - e^{2x}}\right|$
19. $y = e^{e^{x^2}}$
20. $y = (e^3)^{x-1}$
21. $y = e^{2x}e^{3x}e^{4x}$
22. $y = e^x + e^{x+e^x}$
23. $F(t) = e^{t^{1/3}} + (e^t)^{1/3}$
24. $g(t) = e^{-t}\tan e^t$
25. $f(x) = (2x + 1)^3 e^{-(1-x)^4}$
26. $f(x) = e^{x\sqrt{x^2+1}}$
27. $f(x) = \dfrac{xe^x}{x + e^x}$
28. $f(x) = xe^{2x}\ln x$
29. $f(x) = \tan^{-1}e^{2x}$
30. $f(x) = x \sec e^{-x}$

In Problems 31–36 use implicit differentiation to find dy/dx.

31. $y = e^{x+y}$
32. $\ln y = x + e^y$
33. $y = \cos e^{xy}$
34. $y = e^{(x+y)^2}$
35. $x + y^2 = e^{x/y}$
36. $e^x + e^y = y$

In Problems 37–40 find the indicated derivative.

37. $y = e^{-4x}$; $\dfrac{d^3y}{dx^3}$
38. $y = \sin e^{2x}$; $\dfrac{d^2y}{dx^2}$
39. $y = \ln(e^x + 1)$; $\dfrac{d^2y}{dx^2}$
40. $y = xe^x$; $\dfrac{d^4y}{dx^4}$

41. Find an equation of the tangent line to the graph of $y = e^x$ at $x = 1$.

42. Find an equation of the tangent line to the graph of $y = \ln(e^x + 1)$ at $x = 0$.

43. Find the slope of the normal line to the graph of $y = (x - 1)e^{-x}$ at $x = 0$.

44. Find the point on the graph of $y = e^x$ at which the tangent line is parallel to $3x - y = 7$.

In Problems 45–48 sketch the graph of the given function.

45. $y = -e^x$
46. $y = 1 + e^{-x}$
47. $y = 2 - e^{-x}$
48. $y = e^x + e^{-x}$

In Problems 49–54 find the relative extrema of each function. Sketch the graph.

49. $y = xe^{-x}$
50. $y = \dfrac{e^x}{x}$