Exercises 5.4

In Exercises 1–14, evaluate each integral using Part 2 of the Fundamental Theorem *and* using NINT. Vary the order in which you do the two evaluations.

1. $\int_0^3 (4 - x^2)\, dx$

2. $\int_0^1 (x^2 - 2x + 3)\, dx$

3. $\int_0^1 (x^2 + \sqrt{x})\, dx$

4. $\int_0^5 x^{3/2}\, dx$

5. $\int_1^{32} x^{-6/5}\, dx$

6. $\int_{-2}^{-1} \frac{2}{x^2}\, dx$

7. $\int_0^{\pi} \sin x\, dx$

8. $\int_0^{\pi} (1 + \cos x)\, dx$

9. $\int_0^{\pi/3} 2\sec^2 x\, dx$

10. $\int_{\pi/6}^{5\pi/6} \csc^2 x\, dx$

11. $\int_{\pi/4}^{3\pi/4} \csc x \cot x\, dx$

12. $\int_0^{\pi/3} 4\sec x \tan x\, dx$

13. $\int_{-1}^{1} (r + 1)^2\, dr$
(*Hint:* Square first.)

14. $\int_9^4 \frac{1 - \sqrt{u}}{\sqrt{u}}\, du$
(*Hint:* Divide first.)

In Exercises 15–18, find the total area of the region between the curve and the x-axis.

15. $y = 2 - x, \quad 0 \le x \le 3$

16. $y = 3x^2 - 3, \quad -2 \le x \le 2$

17. $y = x^3 - 3x^2 + 2x, \quad 0 \le x \le 2$

18. $y = x^3 - 4x, \quad -2 \le x \le 2$

In Exercises 19–24, can Theorem 4 be used to evaluate the integral? If it can, then find the value. If it cannot, then explain why not.

19. $\int_{-2}^{3} \frac{x^2 - 1}{x + 1}\, dx$

20. $\int_0^5 \frac{9 - x^2}{3x - 9}\, dx$

21. $\int_0^{2\pi} \tan x\, dx$

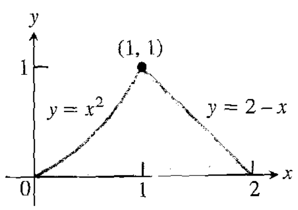
22. $\int_0^2 \frac{x + 1}{x^2 - 1}\, dx$

23. $\int_{-1}^{2} \frac{\sin x}{x}\, dx$

24. $\int_{-2}^{3} \frac{1 - \cos x}{x^2}\, dx$

In Exercises 25–28, find the area of the shaded region.

25.



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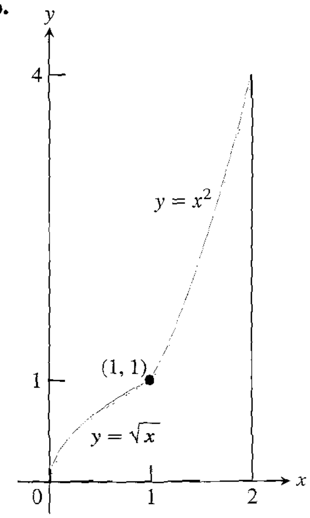

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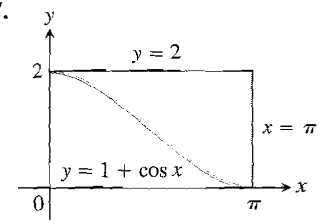

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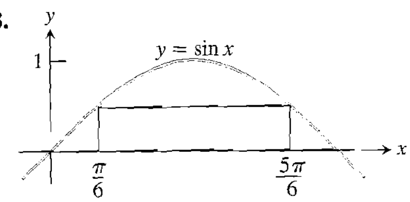


In Exercises 29–32, determine an explicit function for $F(x) = \int_a^x f(t)\, dt$ in terms of x. GRAPH $y = F(x)$ and $y =$ NINT $(f(t), 0, x)$ in the same viewing window. Compare values at $x = 0.5, 1, 1.5, 2,$ and 5.

29. $\int_0^x (t - 2)\, dt$

30. $\int_0^x (t^3 + 1)\, dt$

31. $\int_0^x (t^2 - 3t + 6)\, dt$

32. $\int_0^x 3\sin t\, dt$

GRAPH in the interval specified in Exercises 33–36. Support with SLOPEFLD.

33. $\int_0^x t^2 \sin t\, dt$ for $-3 \le x \le 3$

34. $\int_0^x \sqrt{1+t^2}\, dt$ for $0 \le x \le 5$

35. $\int_0^x 5e^{-0.3t^2}\, dt$ for $0 \le x \le 5$

36. $\int_0^x t \sin(t^3)\, dt$ for $0 \le x \le \pi$

In Exercises 37 and 38, let $F(x) = \int_0^x f(t)\, dt$. GRAPH $y =$ NDER $(F(x), x)$, and compare with the graph of $y = f(x)$.

37. $f(x) = 4 - x^2$ for $-5 \le x \le 5$

38. $f(x) = x \sin x$ for $0 \le x \le 2\pi$

In Exercises 39 and 40, find K so that $\int_a^x f(t)\, dt + K = \int_b^x f(t)\, dt$.

39. $f(x) = x^2 - 3x + 1;\ a = -1;\ b = 2$

40. $f(x) = \sin^2 x;\ a = 0;\ b = 2$

Exercises 41 and 42 involve the power of visualization. In these exercises, solve for x.

41. $\int_0^x e^{-t^2}\, dt = 0.6$ **42.** $\int_0^x \frac{\sin t}{t}\, dt = 1.8$

(*Note*: We assume that the integrand is the continuous extension of $y = (\sin t)/t$.)

Find dy/dx in Exercises 43–46.

43. $y = \int_0^x \sqrt{1+t^2}\, dt$ **44.** $y = \int_1^x \frac{1}{t}\, dt,\ x > 0$

45. $y = \int_0^{\sqrt{x}} \sin(t^2)\, dt$ **46.** $y = \int_0^{2x} \cos t\, dt$

Each of the following functions solves one of the initial value problems in Exercises 47–50. Which function solves which problem?

a) $y = \int_1^x \frac{1}{t}\, dt - 3$ **b)** $y = \int_0^x \sec t\, dt + 4$

c) $y = \int_{-1}^x \sec t\, dt + 4$ **d)** $y = \int_\pi^x \frac{1}{t}\, dt - 3$

47. $\frac{dy}{dx} = \frac{1}{x},\ y(\pi) = -3$ **48.** $y' = \sec x,\ y(-1) = 4$

49. $y' = \sec x,\ y(0) = 4$ **50.** $y' = \frac{1}{x},\ y(1) = -3$

51. For what value of x is

$$\int_a^x f(t)\, dt$$

sure to be zero?

52. Suppose $\int_1^x f(t)\, dt = x^2 - 2x + 1$. Find $f(x)$. (*Hint:* Differentiate both sides of the equation with respect to x.)

53. Find $f(4)$ if $\int_0^x f(t)\, dt = x \cos \pi x$.

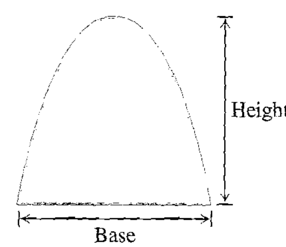
54. Find the linearization of

$$f(x) = 2 + \int_0^x \frac{10}{1+t}\, dt$$

at $x = 0$.

55. Show that if k is a positive constant, then the area between the x-axis and one arch of the curve $y = \sin kx$ is always $2/k$.

56. *Archimedes' area formula for parabolas.* Archimedes (287–212 B.C.), inventor, military engineer, physicist, and the greatest mathematician of classical times, discovered that the area under a parabolic arch like the one shown here is always two-thirds the base times the height.



a) Find the area under the parabolic arch

$$y = 6 - x - x^2, \quad -3 \le x \le 2.$$

b) Find the height of the arch. (Where does y have its maximum value?)

c) Show that the area is two-thirds the base times the height.

57. *Cost from marginal cost.* The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find (a) $c(100) - c(1)$, the cost of printing posters 2–100; (b) $c(400) - c(100)$, the cost of printing posters 101–400.

58. *Revenue from marginal revenue.* Suppose that a company's marginal revenue from the manufacture and sale of eggbeaters is

$$\frac{dr}{dx} = 2 - 2/(x+1)^2,$$

where r is measured in thousands of dollars and x in thousands of units. How much money should the company expect from a production run of $x = 3$ thousand eggbeaters? To find out, integrate the marginal revenue from $x = 0$ to $x = 3$.