Calculus

Mid-Term Review

Meltz

Name:_____

1. If $f(x)=3-x^2$, *find*:
   a) f(3)
   b) f(-1)
   c) $f(2+\Delta x)$

2. Let $f(x)=\dfrac{5}{x-1}$ and $g(x)=x^4$
   a) Find $f\big(g(x)\big)$
   b) Find all values of x for which $f\big(g(x)\big)$ is discontinuous.

3. Find the values of x for all points on the graph of $f(x)=x^3-2x^2+5x-16$ at which the slope of the tangent line is 4.

4. Differentiate: $y=\dfrac{2x}{1-3x^2}$

5. Find $\lim\limits_{x\to -3}\left(-2x^2+1\right)$

   (a) 37 (b) 19 (c) -17

   (d) $\pm\sqrt{2}$ (e) None of these

6. Find $\lim\limits_{x\to -1}\dfrac{x^2+2x+3}{x^2+1}$

   (a) 0 (b) 1 (c) ∞

   (d) Does Not Exist (e) None of these

7. *If* $\lim\limits_{x\to c} f(x)=-\dfrac{1}{2}$ and $\lim\limits_{x\to c} g(x)=\dfrac{2}{3}$, find $\lim\limits_{x\to c}\left[f(x)g(x)\right]$

   (a) $\dfrac{1}{6}$ (b) $-\dfrac{1}{3}$ (c) 1

   (d) Does Not Exist (e) None of these

8. Find $\lim_{x\to -1}\dfrac{x^2-5x-6}{x+1}$

(a) 0 (b) -7 (c) $-\infty$

(d) ∞ (e) None of these

9 Find $\lim_{x\to 2}\dfrac{x-2}{|x-2|}$

(a) 0 (b) 1 (c) 2

(d) Does Not Exist (e) None of these

10. Find $\lim_{x\to 0}\dfrac{\sqrt{x+9}-3}{x}$

(a) 0 (b) 1 (c) ∞

(d) $\dfrac{1}{3}$ (e) None of these

11. Find $\lim_{x\to 3^+}\sqrt{2x-5}$

(a) 1 (b) 0 (c) $2i$

(d) Does Not Exist (e) None of these

12. At which values of x is $f(x)=\dfrac{x-4}{(x-2)(x+1)}$ discontinuous?

(a) 4 (b) -1, 2, 4 (c) -1, 2

(b) -1, 2, 4, -2 (e) None of these

13. Find $\lim_{x\to 0}\left(2+\frac{5}{x^2}\right)$

(a) 7 (b) 2 (c) ∞

(d) $-\infty$ (e) None of these

14. $f(x)$ decreases without bound as x approaches what value from the right?

$$f(x)=\frac{4}{(x-3)(5-x)}$$

(a) $x\to 5^+$ (b) $x\to 3^-$ (c) $x\to 5^-$

(d) $x\to 3^+$ (e) None of these

15. If $f(x)=-x^2+x$, which of the following will calculate the derivative of $f(x)$?

(a) $\lim_{\Delta x\to 0}\frac{\left(-x^2+x+\Delta x\right)-\left(-x^2+x\right)}{\Delta x}$

(b) $\lim_{\Delta x\to 0}\frac{\left[-\left(x+\Delta x\right)^2+\left(x+\Delta x\right)\right]-\left(-x^2+x\right)}{\Delta x}$

(c) $\frac{\left[-\left(x+\Delta x\right)^2+\left(x+\Delta x\right)\right]-\left(-x^2+x\right)}{\Delta x}$

(d) $\frac{\left(-x^2+x+\Delta x\right)-\left(-x^2+x\right)}{\Delta x}$

(e) None of these

16. *If* $f(1)=4$ *and* $f'(x)=2$, find an equation of the tangent line when $x=1$

(a) $y-4=2(x-1)$ (b) $y+4=2(x+1)$

(c) $y-1=4(x-2)$ (d) $y-2=4(x-1)$

(e) None of these

17. *Find* $f'(x)$

$$f(x)=3x^4-6x^3+3x-2$$

(a) $f'(x)=3x^3-6x^2+3$ (b) $f'(x)=12x^3-18x^2+3x-2$

(c) $f'(x)=12x^3-18x^2+3$ (d) $f'(x)=3x^4-6x^3+3x$

(e) None of these

18. *Find* $f'(x)$

$$f(x)=\frac{x^2-3x}{x^2}$$

(a) $f'(x)=\frac{2x-3}{x^2}$ (b) $f'(x)=\frac{2x-3}{2x}$

(c) $f'(x)=1-\frac{3}{x}$ (d) $f'(x)=\frac{3}{x^2}$

(e) None of these

19. Find the instantaneous rate of change of w with respect to z if $w=\frac{7}{3z^2}$

(a) $\frac{7}{6z}$ (b) $\frac{14}{3}z$ (c) $-\frac{14}{3z}$

(d) $-\frac{14}{3z^3}$ (e) None of these

20. Find an equation for the tangent line to the graph of $f(x) = -2x^2 + 2x + 3$ at the point where x = 1.

(a) $y = -4x + 2$ (b) $2x + y - 1 = 0$

(c) $y = -4x^2 + 2x + 1$ (d) $2x + y = 5$

(e) None of these

21. Suppose the position equation for a moving object is given by $s(t) = 3t^2 - 2t + 5$ where s is measured in meters and t is measured in seconds. Find the velocity of the object when t = 2.

(a) $13\ m/\sec$ (b) $6\ m/\sec$ (c) $10\ m/\sec$

(d) $14\ m/\sec$ (e) None of these

22. Differentiate: $f(x) = -x + \tan x$

(a) $-1 + \tan^2 x$ (b) $\sec^2 x$ (c) $\tan^2 x$

(d) $-1 + \tan x$ (e) None of these

23. *Find* $\frac{dy}{dx}$ *for* $y = \sqrt{x}(3x - 1)$

(a) $\frac{9x - 1}{2\sqrt{x}}$ (b) $\frac{9}{2}\sqrt{x} - 1$ (c) $3\sqrt{x}$

(d) $\frac{3}{2\sqrt{x}}$ (e) None of these

24. Differentiate: $y = \csc^2\theta + \cot^2\theta$

(a) $\cot\theta + \csc^4\theta$ (b) 0

(c) $-4\csc^2\theta\ \cot\ \theta$ (d) $-\csc^2\theta\left(\csc^2\theta + \cot^2\theta\right)$

(e) None of these

25. *Find* y' *if* $x^2 + y^2 = 2xy$

(a) $\dfrac{x}{1-y}$ (b) $\dfrac{y+x}{y-x}$ (c) 1

(d) $-\dfrac{x}{y}$ (e) None of these

26. Find all critical numbers:

$$f(x) = \left(9 - x^2\right)^{\frac{3}{5}}$$

(a) 0 (b) 3 (c) -3, 3

(d) -3, 0, 3 (e) None of these

27. Find all extrema in the interval $[0, 2\pi]$ if $y = x - \cos x$

(a) $\left(-1, -1 + \dfrac{3\pi}{2}\right)$ (b) $(\pi, \pi + 1)$ (c) $(-1, 0)$

(d) $\left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right)$ (e) None of these

28. Find the absolute maximum and absolute minimum of f on $(-1, 2]$

$f(x) = \dfrac{x^3 - x^2 - 3x - 1}{x + 1}$ (Use graphing calculator)

(a)	Maximum:	(-1 , 2)	Minimum:	(2 , -1)
(b)	Maximum:	None	Minimum:	(1 , -2)
(c)	Maximum:	(-1 , 2)	Minimum:	(1 , -2)
(d)	Maximum:	None	Minimum:	(2 , -1)
(e)	None of these			

29. Find all open intervals on which $f(x)=\dfrac{x}{x^2+x-2}$ is decreasing.

(a) $(-\infty,\infty)$ (b) $(-\infty,0)$

(c) $(-\infty,-2)$ *and* $(1,\infty)$ (d) $(-\infty,-2),(-2,1)$ *and* $(1,\infty)$

(e) None of these

30. Find all intervals on which the function is concave upward:

$$f(x)=\frac{x-1}{x+3}$$

(a) $(-\infty,\infty)$ (b) $(-\infty,-3)$ (c) $(1,\infty)$

(d) $(-3,\infty)$ (e) None of these

31. Find all points of inflection:

$$f(x)=x^3-12x$$

(a) $(0,0),(\pm\sqrt{12},0)$ (b) $(0,0)$

(c) $(-2,0),(2,0)$ (e) $(2,-16),(-2,16)$

(e) None of these

32. Find the average rate of change of $f(x)=3x^2-2$ on the interval $[0,2]$.

(a) 1 (b) 5 (c) 6

(d) 2 (e) None of these

33. Find the derivative of $f(x)=2x^2+8x-7$ using the definition of derivative.