y's) and compare the relative growth rates of the functions. In general, how does the exponential function compare to polynomials?

4. Graphically compare and contrast the functions $x^{1/2}$, $x^{1/3}$, $x^{1/4}$ and $\ln x$ for $x > 1$. Sketch the graphs for large x and compare the relative growth rates of the functions. In general, how does $\ln x$ compare to $\sqrt[n]{x}$?

In exercises 1–24, find the derivative of the function.

1. $f(x) = x^3 e^x$
2. $f(x) = e^{2x} \cos 4x$
3. $f(x) = x + 2^x$
4. $f(x) = x4^{3x}$
5. $f(x) = 2e^{4x+1}$
6. $f(x) = (1/e)^x$
7. $f(x) = (1/3)^{x^2}$
8. $f(x) = 4^{-x^2}$
9. $f(x) = 4^{-3x+1}$
10. $f(x) = (1/2)^{1-x}$
11. $f(x) = \dfrac{e^{4x}}{x}$
12. $f(x) = \dfrac{x}{e^{6x}}$
13. $f(x) = \ln 2x$
14. $f(x) = \ln \sqrt{8x}$
15. $f(x) = \ln(x^3 + 3x)$
16. $f(x) = x^3 \ln x$
17. $f(x) = \ln(\cos x)$
18. $f(x) = e^{\sin 2x}$
19. $f(x) = \sin[\ln(\cos x^3)]$
20. $f(x) = \ln(\sin x^2)$
21. $f(x) = \dfrac{\sqrt{\ln x^2}}{x}$
22. $f(x) = \dfrac{e^x}{2^x}$
23. $f(x) = \ln(\sec x + \tan x)$
24. $f(x) = \sqrt[3]{e^{2x} x^3}$

In exercises 25–30, find an equation of the tangent line to $y = f(x)$ at $x = 1$.

25. $f(x) = 3e^x$
26. $f(x) = 2e^{x-1}$
27. $f(x) = 3^x$
28. $f(x) = 2^x$
29. $f(x) = x^2 \ln x$
30. $f(x) = 2\ln x^3$

In exercises 31–34, the value of an investment at time t is given by $v(t)$. Find the instantaneous percentage rate of change.

31. $v(t) = 100\, 3^t$
32. $v(t) = 100\, 4^t$
33. $v(t) = 100\, e^t$
34. $v(t) = 100\, e^{-t}$

35. A bacterial population starts at 200 and triples every day. Find a formula for the population after t days and find the percentage rate of change in population.

36. A bacterial population starts at 500 and doubles every four days. Find a formula for the population after t days and find the percentage rate of change in population.

37. An investment of A dollars receiving $100r$ percent (per year) interest compounded continuously will be worth $f(t) = Ae^{rt}$ dollars after t years. APY can be defined as $[f(1) - A]/A$, the relative increase of worth in one year. Find the APY for the following interest rates:

(a) 5% (b) 10% (c) 20%
(d) 100 ln 2% (e) 100%

38. Determine the interest rate needed to obtain an APY of

(a) 100% (b) 10%

In exercises 39–44, use logarithmic differentiation to find the derivative.

39. $f(x) = x^{\sin x}$
40. $f(x) = x^{4-x^2}$
41. $f(x) = (\sin x)^x$
42. $f(x) = (x^2)^{4x}$
43. $f(x) = x^{\ln x}$
44. $f(x) = x^{\sqrt{x}}$

45. The motion of a spring is described by $f(t) = e^{-t} \cos t$. Compute the velocity at time t. Graph the velocity function. When is the velocity zero? What is the position of the spring when the velocity is zero?

46. The motion of a spring is described by $f(t) = e^{-2t} \sin 3t$. Compute the velocity at time t. Graph the velocity function. When is the velocity zero? What is the position of the spring when the velocity is zero?

47. In exercise 45, graphically estimate the value of $t > 0$ at which the maximum velocity is reached.

48. In exercise 46, graphically estimate the value of $t > 0$ at which the maximum velocity is reached.

In exercises 49–52, involve the hyperbolic sine and hyperbolic cosine functions: $\sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

49. Show that $\dfrac{d}{dx}(\sinh x) = \cosh x$ and $\dfrac{d}{dx}(\cosh x) = \sinh x$.

50. Find the derivative of the hyperbolic tangent function: $\tanh x = \dfrac{\sinh x}{\cosh x}$.

51. Show that both $\sinh x$ and $\cosh x$ have the property that $f''(x) = f(x)$.

52. Find the derivative of (a) $f(x) = \sinh(\cos x)$ and (b) $f(x) = \cosh(x^2) - \sinh(x^2)$.

53. Find the value of a such that the tangent to $\ln x$ at $x = a$ is a line through the origin.

54. Find the value of a such that the tangent to e^x at $x = a$ is a line through the origin. Compare the slopes of the lines in exercises 53 and 54.

In exercises 55–58, use a CAS or graphing calculator.

55. Find the derivative of $f(x) = e^{\ln x^2}$ on your CAS. Compare its answer to $2x$. Explain how to get this answer and your CAS's answer, if it differs.