KEY

MAX/MIN & CONCAVITY Review Sheet

Find all asymptotes, all local min's and max's, all inflection points and where the graph is rising/falling & concave up/concave down.

1) y = x + sinx in the interval [-2π , 2π]

$f'(x) = 1 + \cos x$

$0 = 1 + \cos x$

$-1 = \cos x$

$x = \pi, -\pi$

$+ \quad + \quad +$

$-\pi \quad \pi$

$f''(x) = -\sin x$

$0 = \sin x$

$x = -2\pi, -\pi, 0, \pi, 2\pi$

$- \quad + \quad - \quad +$

$-2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi$

rising on $[-2\pi, 2\pi]$

concave up on $[-\pi, 0]$ and $[\pi, 2\pi]$

concave down on $[-2\pi, -\pi]$ and $[0, \pi]$

inflection pts at $(-\pi, -\pi)$, $(0, 0)$, (π, π)

no local min's or max's

2) y = x³ - 2x² + x

$f'(x) = 3x^2 - 4x + 1$

$(3x - 1)(x - 1)$

$x = \frac{1}{3}, 1$

$+ \quad - \quad +$

$\frac{1}{3} \quad 1$

$f''(x) = 6x - 4$

$0 = 6x - 4$

$x = \frac{2}{3}$

$f''(x)$: $- \quad +$

$\frac{2}{3}$

local max at $\left(\frac{1}{3}, \frac{4}{27}\right)$

local min at $(1, 0)$

rising on $\left(-\infty, \frac{1}{3}\right]$ and $[1, \infty)$

falling on $\left[\frac{1}{3}, 1\right]$

concave down on $\left(-\infty, \frac{2}{3}\right]$

concave up on $\left[\frac{2}{3}, \infty\right)$

inflection point at $\left(\frac{2}{3}, \frac{2}{27}\right)$

3) $y = 2x^3 - x^2 - 14x - 12$

$f'(x) = 6x^2 - 2x - 14$

$0 = 2(3x^2 - x - 7) = 0$

$x = -1.37,\ 1.70$

+   −   +

−1.37   1.70

$f''(x) = 12x - 2$

$0 = 12x - 2$

$x = \frac{1}{6}$

−   +

$\frac{1}{6}$

rising on $(-\infty, -1.37]$ and $[1.70, \infty)$

falling on $[-1.37, 1.70]$

local max at $(-1.37, 9.16)$

local min at $(1.70, -28.86)$

concave down on $(-\infty, \frac{1}{6}]$

concave up on $[\frac{1}{6}, \infty)$

inflection point at $(\frac{1}{6}, -14.35)$

4) $y = \dfrac{x^2 - 4}{x - 1}$

$u = x^2 - 4$ $v = x - 1$

$du = 2x$ $dv = 1$

$f'(x) = \dfrac{2x(x-1) - (x^2-4)}{(x-1)^2} = \dfrac{2x^2 - 2x - x^2 + 4}{} \quad \boxed{\dfrac{x^2 - 2x + 4}{(x-1)^2}}$

$x^2 - 2x + 4 = 0$

$x = \varnothing$ (Quad formula gives imaginary roots)

undefined at $x = 1$

$f'(x)$ +   + (at 1)

$\boxed{\text{increasing on } (-\infty, \infty) \text{ no local min's or max's}}$

$f''(x) = \longrightarrow \dfrac{[2x-2(x-1)^2] - [2(x-1)(x^2-2x+4)]}{(x-1)^4} = \dfrac{(2x-2)(x-1) - 2(x^2-2x+4)}{(x-1)^3}$

$u = x^2 - 2x + 4$ $v = (x-1)^2$

$du = 2x - 2$ $dv = 2(x-1)$

$= \dfrac{2x^2 - 2x - 2x + 2 \quad -2x^2 + 4x - 8}{(x-1)^3} = \dfrac{-6}{(x-1)^3}$ undefined at $x = 1$

$f''(x)$ +   − (at 1)

concave up on $(-\infty, 1]$

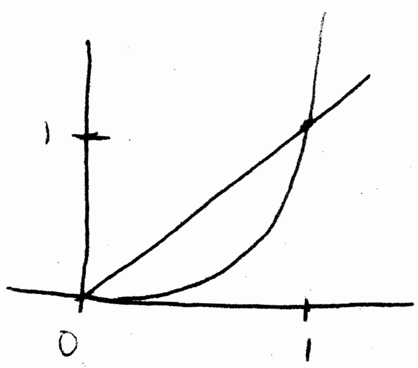
~~inflection point at~~

Vertical asymptote at $x = 1$

Areas & Volumes Review

I - Find the area of the region bounded by the given curves:

1) $y = x^2$, $y = x$



$$\int_0^1 x - x^2 \, dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} \Big]_0^1 = \left[\left(\frac{1}{2} - \frac{1}{3}\right) - (0)\right] = \frac{1}{6}$$

2) $x + y = 0$, $x = y^2 + 3y$

$y = -x$
or
$x = -y$

x	y
0	0
4	1
10	2
-2	-1
-2	-2
18	3

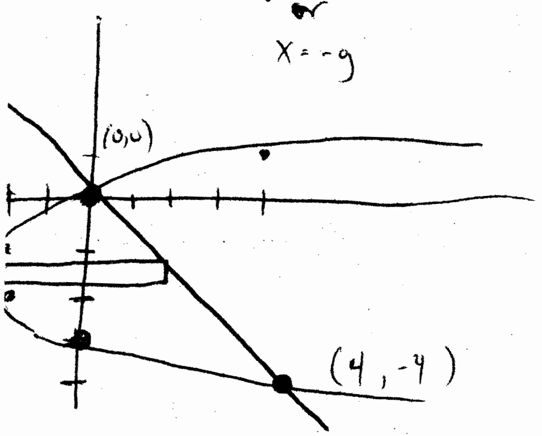
find pts of intersection

$y^2 + 3y = -y$

$y^2 + 4y = 0$

$y(y+4) = 0$

$y = 0$ and $y = -4$

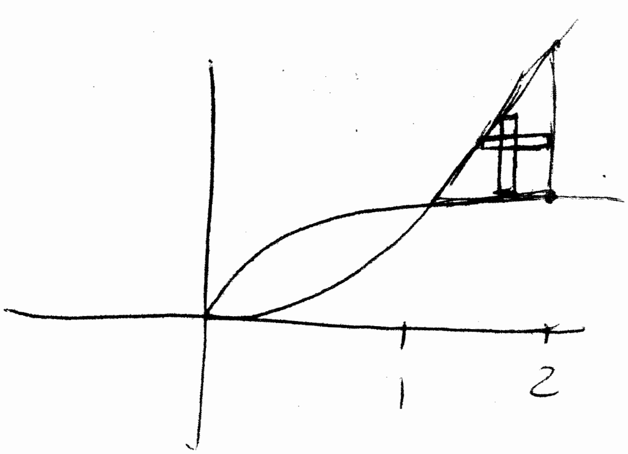


$$\int_{-4}^{0} -y - (y^2 + 3y) \, dy = \int_{-4}^{0} -y^2 - y - 3y \, dy = \frac{-y^3}{3} - \frac{y^2}{2} - \frac{3}{2}y^2 \Big]_{-4}^{0}$$

$$= (0) - \left[\frac{-(-4)^3}{3} - \frac{(-4)^2}{2} - \frac{3}{2}(-4)^2\right] = 0 - \left[\frac{64}{3} - 8 - 24\right]$$

$$0 - \left(-\frac{32}{3}\right) = \frac{32}{3}$$

3) $y = \sqrt{x}$, $y = x^2$, $x = 2$



$$\int_1^2 x^2 - \sqrt{x} \, dx = \frac{x^3}{3} - \frac{2}{3}x^{\frac{3}{2}} \Big]_1^2$$

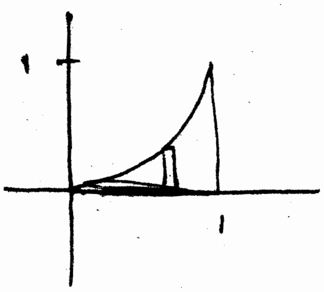
$$= \left[\frac{2^3}{3} - \frac{2}{3}(2)^{\frac{3}{2}}\right] - \left[\frac{1}{3} - \frac{2}{3}(1)^{\frac{3}{2}}\right]$$

$$\left[.78104\right] - \left[-\frac{1}{3}\right]$$

$$= \boxed{1.114}$$

II - Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

4) $y = x^2$, $x = 1$, $y = 0$; about the x-axis

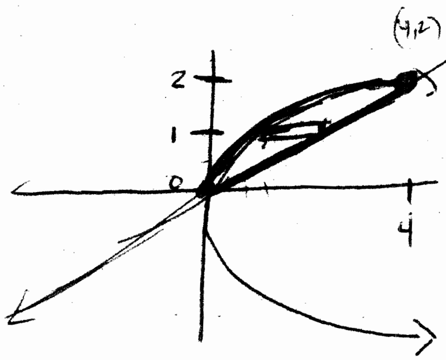


$$V = \pi \int r^2\, dr$$

$$V = \pi \int_0^1 (x^2)^2\, dx = \pi \int_0^1 x^4\, dx = \pi \left[\frac{x^5}{5}\right]_0^1$$

$$= \pi\left(\frac{1}{5} - 0\right) = \frac{\pi}{5}$$

5) $y^2 = x$, $x = 2y$; about the y-axis



Points of Intersection

$y^2 = 2y$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y=0$ / $y=2$

$$V = \pi \int_0^2 (2y)^2 - (y^2)^2\, dy$$

$$\pi \int_0^2 4y^2 - y^4\, dy$$

$$= \pi \left[\frac{4y^3}{3} - \frac{y^5}{5}\right]_0^2$$

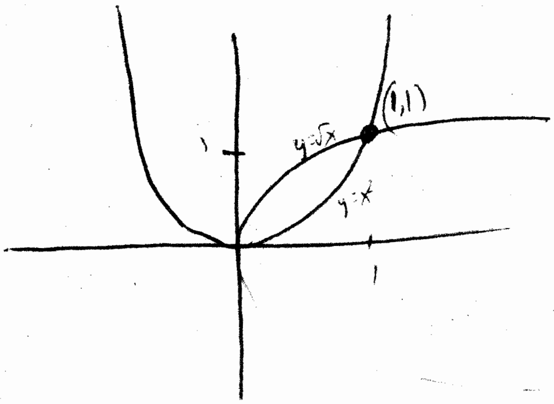
$$= \pi \left[\frac{4(2^3)}{3} - \frac{2^5}{5}\right]$$

$$\pi \left[\frac{32}{3} - \frac{32}{5}\right] = \frac{64\pi}{15}$$

6) $y = x^2$, $x = y^2$; about the x-axis

$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \; dx$$

$x^2 =$



$$V = \pi \int_0^1 x - x^4 \; dx$$

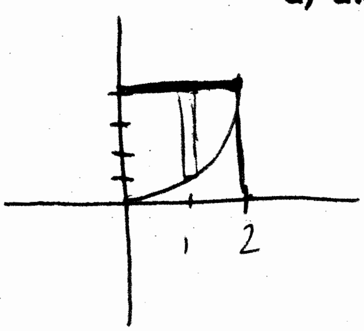
$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{5} \right]$$

$$= \frac{3\pi}{10}$$

7) Find the volumes of the solids obtained by rotating the region bounded by the curves $y = 4$ and $y = x^2$ about the following lines:

a) the ~~[illegible]~~is X-AXIS



$$V = \pi \int_0^2 4^2 - (x^2)^2 \; dx$$

$$V = \pi \int_0^2 16 - x^4 \; dx$$

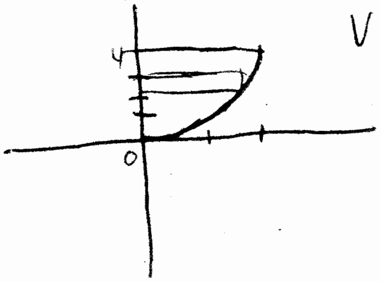
$$\pi \left[16x - \frac{x^5}{5} \right]_0^2 = \pi \left[32 - \frac{32}{5} \right]$$

$$\frac{128\pi}{5}$$

b) the ~~x-axis~~ y-Axis

$y = x^2$

$x = \sqrt{y}$



$$V = \pi \int_0^4 (\sqrt{y})^2 \; dy$$

$$V = \pi \int_0^4 y \; dy$$

$$\pi \left[\frac{y^2}{2} \right]_0^4 = \pi \left(\frac{4^2}{2} \right) = 8\pi$$

Derivative/Tangent Line Review

For each of the following:

A) Find the equation of the tangent line for each of the following functions at each given value of x.

B) Draw the graph of the function and the tangent line with your calculator and copy graph.

C) Indicate in your graph the point of intersection between the curve and the tangent line.

(1, 6)

1) $y = 3x^2 - 4x + 7$ at $x = 1$

A) $y' = 6x - 4$

$f'(1) = 2$

slope is 2

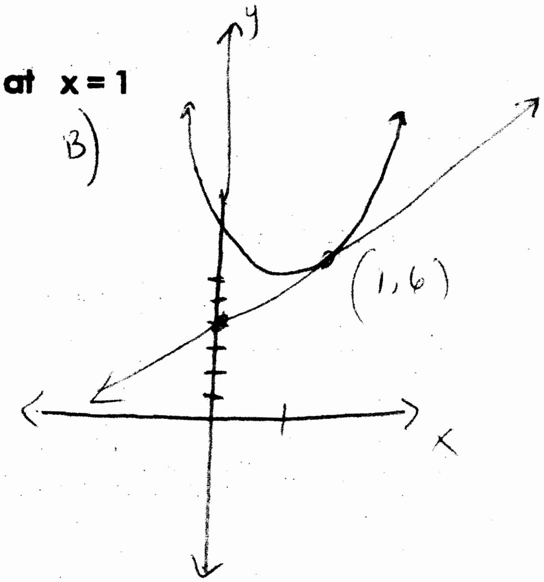
$y - y_1 = m(x - x_1)$

$y - 6 = 2(x - 1)$

$y - 6 = 2x - 2$

$y = 2x + 4$

B)



(2, −3)

2) $y = -x^2 + 2x - 3$ at $x = 2$

$f'(x) = -2x + 2$

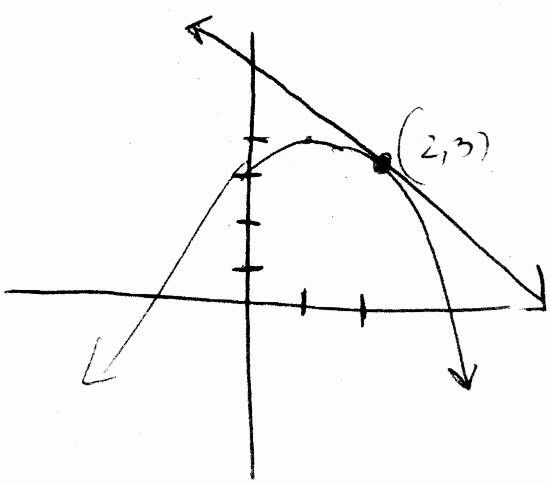
$f'(2) = -2(2) + 2 = -2$

$y - y_1 = m(x - x_1)$

$y + 3 = -2(x - 2)$

$y + 3 = -2x + 4$

$\boxed{y = -2x + 1}$



(0,3)

3) $y = -4x^2 + 3$ at $x = 0$

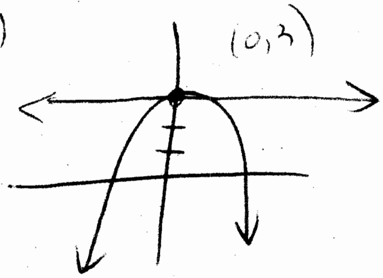
$f'(x) = -8x$

$f'(0) = 0$

$y - 3 = 0(x - 0)$

$y = 3$

(0,3)



(0,4)

4) $y = 3x^2 - x + 4$ at $x = 0$

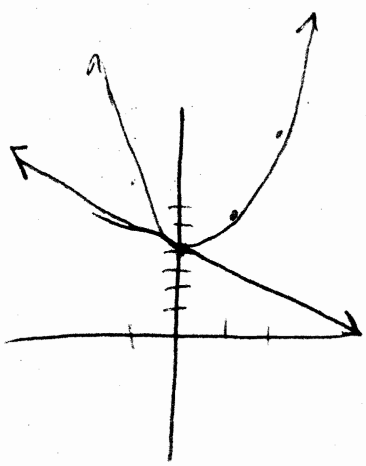
$f'(x) = 6x - 1$

$f'(0) = -1$

$y - 4 = -1(x - 0)$

$y - 4 = -x$

$y = -x + 4$



(0,4)

(0,0)

5) $y = -x^2$ at $x = 0$

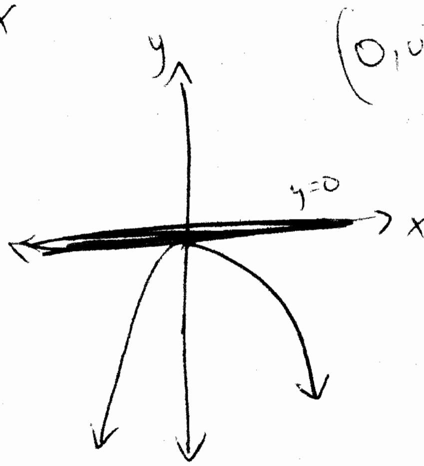
$f'(x) = -2x$

$f'(0) = 0$

$y - 0 = 0(x - 0)$

$y = 0$

(0,0)



Limits Review

1) $\lim_{x \to 0} \frac{\sin x}{x+2} = \frac{0}{2} = 0$

2) $\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{0}{0} \overset{L}{=} \frac{-1}{2x} = \frac{+1}{2x} = \frac{1}{2(1)} = \frac{1}{2}$

3) $\lim_{x \to \infty} \frac{x+1}{x} = \frac{1}{1} = 1$ (think about EBA's!)

OR $\frac{\infty}{\infty} \overset{L}{=} \frac{1}{1} = 1$

4) $\lim_{x \to -1} \frac{x+3}{x^2+3x+1} = \frac{-1+3}{1-3+1} = \frac{+2}{-1} = -2$

5) $\lim_{x \to -2} \frac{x+2}{x^2-4} = \frac{0}{0} \overset{L}{=} \frac{1}{2x} = \frac{1}{2(-2)} = -\frac{1}{4}$

6) $\lim_{x \to 2} \frac{2x-4}{x^3-2x^2} = \frac{0}{0} \overset{L}{=} \frac{2}{3x^2-4x} = \frac{2}{12-8} = \frac{2}{4} = \frac{1}{2}$

7) $\lim\limits_{x \to 1} \frac{\ln x}{1-x}$ L $= \frac{0}{0} = \frac{\frac{1}{x}}{-1} = -\frac{1}{x} = -\frac{1}{1} = -1$

8) $\lim\limits_{x \to 1} \frac{\ln x}{e^x - e}$ L $= \frac{0}{0} = \frac{\frac{1}{x}}{e^x} = \frac{1}{x} \cdot \frac{1}{e^x} = \frac{1}{1 \cdot e^1} = \frac{1}{e}$

9) $\lim\limits_{x \to 0} \frac{\sin x}{2x^2 - x} = \lim\limits_{x \to 0} \frac{\sin x}{x(2x-1)} = \frac{\sin x}{x} \cdot \frac{1}{(2x-1)}$

$= 1 \cdot \frac{1}{2(0)-1} = -1$

10) $\lim\limits_{x \to -1} \frac{x^2 - 1}{x} = \frac{0}{-1} = 0$

11) $\lim\limits_{t \to 1} \frac{t^2 - 3t + 2}{t^2 - 1}$ L $= \frac{0}{0} = \frac{2t - 3}{2t} = \frac{2(1) - 3}{2(1)} = \frac{-1}{2} = -\frac{1}{2}$

12) $\lim\limits_{x \to -2} \frac{x + 2}{2^x} = \frac{0}{2^{-2}} = \frac{0}{\frac{1}{4}} = \frac{0}{1} = 0$

13) $\lim\limits_{x \to 1} \frac{\ln x}{x^2 - 1}$ L $= \frac{0}{0} = \frac{\frac{1}{x}}{2x}$

$\frac{1}{x} \div 2x$

$\frac{1}{x} \cdot \frac{1}{2x} = \frac{1}{2x^2} = \frac{1}{2}$